QUIZ 2 SOLUTIONS

(a). $M_{\underset{\sim}{x}}(\underset{\sim}{t}) = \left(p_1 e^{t_1} + p_2 e^{t_2} + \cdots + p_r e^{t_r}\right)^n$

THEREFORE,

$$M_{x_1}(t_1) = M_{\underset{\sim}{x}}(t_1, 0, 0, \ldots, 0) = \Big(p_1 e^{t_1} + \underbrace{p_2 + \cdots + p_r}_{1-p_1}\Big)^n$$

$$= \left(p_1 e^{t_1} + 1 - p_1\right)^n \quad \therefore X_1 \sim b(n, p_1)$$

(b). $M_1(t) = \dfrac{\partial M_{\underset{\sim}{x}}(\underset{\sim}{t})}{\partial t_1} = np_1 e^{t_1}\left(p_1 e^{t_1} + p_2 e^{t_2} + \cdots + p_r e^{t_r}\right)^{n-1}$

$$\therefore M_1(\underset{\sim}{0}) = np_1$$

FOR THE VARIANCE WE USE THE COROLLARY:

$$\psi(\underset{\sim}{t}) = \ln M_{\underset{\sim}{x}}(\underset{\sim}{t}) = n \ln\left(p_1 e^{t_1} + \cdots + p_r e^{t_r}\right)$$

$$\psi_1(t) = \frac{\partial \psi(\underset{\sim}{t})}{\partial t_1} = \frac{np_1 e^{t_1}}{p_1 e^{t_1} + \cdots + p_r e^{t_r}}$$

$$\psi_{11}(t) = \frac{np_1 e^{t_1}\left(p_1 e^{t_1} + \cdots + p_r e^{t_r}\right) - p_1 e^{t_1} np_1 e^{t_1}}{\left(p_1 e^{t_1} + \cdots + p_r e^{t_r}\right)^2}$$

$$\psi_{11}(\underset{\sim}{0}) = np_1 - np_1^2 = np_1(1 - p_1)$$

(c). USE THE COROLLARY:

$$\psi_i(\underline{t}) = \frac{n p_i e^{t_i}}{p_1 e^{t_1} + \cdots + p_r e^{t_r}}$$

$$\psi_{ij}(\underset{\sim}{t}) = \frac{-p_j e^{t_j} n p_i e^{t_i}}{\left(p_1 e^{t_1} + \cdots + p_r e^{t_r}\right)^n}$$

$$\psi_{ij}(\underset{\sim}{0}) = -n p_i p_j$$

WHEN X_i INCREASES X_j WILL DECREASE,

SINCE $X_1 + X_2 + \cdots + X_r = n$.

(d). (1). $M_{X+Y}(t) = M_X(t) M_Y(t) = e^{2\mu t + t^2\sigma^2}$

$M_{X-Y}(s) = M_X(s) M_Y(-s) = e^{s^2\sigma^2}$

(2). $M_{X+Y,X-Y}(t,s) = E e^{t(X+Y) + s(X-Y)}$

$$= E e^{X(t+s)+Y(t-s)}$$

$$= M_X(t+s)\, M_Y(t-s)$$

$$= e^{\mu(t+s)+\frac{1}{2}(t+s)^2\sigma^2} \cdot e^{\mu(t-s)+\frac{1}{2}(t-s)^2\sigma^2}$$

$$= e^{2\mu t+t^2\sigma^2}\, e^{t^2\sigma^2} = M_{X+Y}(t) \cdot M_{X-Y}(s)$$

(3). YES, THEY ARE INDEPENDENT BECAUSE

$$M_{X+Y,\,X-Y}(t,s) = M_{X+Y}(t) \cdot M_{X-Y}(s).$$