University of California, Los Angeles
Department of Statistics

Statistics 100C **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1
Data have been collected for 19 observations of two variables, y and x, in order to run a regression of y on x. You are given that $s_y = 10$, $\sum_{i=1}^{19}(y_i - \hat{y}_i)^2 = 180$.

a. Compute the proportion of the variation in y that can be explained by x. [Ans. 0.90]

b. Compute the standard error of the estimate (s_e). [Ans. 10.59]

EXERCISE 2
Data on y and x were collected to run a regression of y on x. The intercept is included. You are given the following: $\bar{x} = 76, \bar{y} = 880, \sum_{i=1}^{n}(x_i - \bar{x})^2 = 6800, \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 14200, r_{xy} = 0.72, s_e = 20.13$.

a. What is the value of $\hat{\beta}_1$? [Ans. 2.088]

b. What is the value of $\hat{\beta}_0$? [Ans. 721.312]

c. What is the value of $\sum_{i=1}^{n}(y_i - \bar{y})^2$? [Ans. 57188]

d. What is the sample size n? [Ans. 70]

EXERCISE 3
Observations on both X and Y are standardized, having estimated means of 0 and standard deviation 1. Show that the fitted line equation has the form $\hat{y}_i = rx_i$, where r is the correlation coefficient between Y and X.

EXERCISE 4
Show that for the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ you can express SSR as

$$SSR = \hat{\beta}_1^2 \sum_{i=1}^{n}(x_i - \bar{x})^2$$

EXERCISE 5
Let F be the F-statistic for the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Show that the F-statistic can be expressed in terms of R^2 as follows:

$$F = \frac{R^2}{1 - R^2}(n - 2).$$

EXERCISE 6
Show that for the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ it is true that

$$E(MSR) = \sigma^2 + \beta_1^2 \sum_{i=n}^{n}(x_i - \bar{x})^2.$$

EXERCISE 7
For the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, suppose we want to test
$H_0 : \beta_0 - 2\beta_1 = 0$
$H_a : \beta_0 - 2\beta_1 \neq 0$
Construct a t-test to test the above hypothesis.

EXERCISE 8
Consider the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Find $var(e_i)$, where $e_i = Y_i - \hat{Y}_i$.