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Statistics 100C **Instructor: Nicolas Christou**

Partial regression

We have seen that if we partition the matrix $\mathbf{X} = (\mathbf{X_1}, \mathbf{X_2})$ and the vector $\boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \end{pmatrix}$ we can obtain the estimate of $\boldsymbol{\beta_2}$ using the following two-step procedure:

A. Run the partial regressions:

1. Regress $\mathbf{Y}$ on $\mathbf{X_1}$ to obtain the residuals $\mathbf{Y}^* = (\mathbf{I} - \mathbf{H_1})\mathbf{Y}$.
2. Regress each column of $\mathbf{X_2}$ on $\mathbf{X_1}$ to obtain the residuals $\mathbf{X_2^*} = (\mathbf{I} - \mathbf{H_1})\mathbf{X_2}$.

B. Obtain the estimate of $\boldsymbol{\beta_2}$ by regressing $\mathbf{Y}^*$ on $\mathbf{X_2^*}$. It is equal to $\hat{\boldsymbol{\beta}}_{\mathbf{2.1}} = (\mathbf{X}^{*\prime}\mathbf{X}^*)^{-1}\mathbf{X}^{*\prime}\mathbf{Y}^*$.

A general note:
The matrix $\mathbf{I} - \mathbf{H}$ is is called the residual maker matrix of the regression of $\mathbf{Y}$ on $\mathbf{X}$. The residual vector is equal to $\mathbf{e} = (\mathbf{I} - \mathbf{H})\mathbf{Y}$.

Now consider the following two special cases:

a. Partition $\mathbf{X} = (\mathbf{1}, \mathbf{X_{(0)}})$ and the vector $\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \boldsymbol{\beta_{(0)}} \end{pmatrix}$. Use partial regression to estimate $\boldsymbol{\beta_{(0)}}$. What is the residual maker matrix in this case?

B. Suppose the model is $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. We then decided to add another single predictor $\mathbf{z}$. So the model is now $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + c\mathbf{z} + \boldsymbol{\epsilon}$. Use partial regression to estimate c.