University of California, Los Angeles
Department of Statistics

Statistics 100C **Instructor: Nicolas Christou**

Standardized residuals and leverage points - example

The rain/wheat data:

```
   rain wheat
1    12   310
2    14   320
3    13   323
4    16   330
5    18   334
6    20   348
7    19   352
8    22   360
9    22   370
10   20   344
11   23   370
12   24   380
13   26   385
14   27   393
15   28   395
16   29   400
17   30   403
18   31   406
19   26   383
20   27   388
21   28   392
22   29   398
23   30   400
24   31   403
25   20   270
26   50   260
```

For this data the variance of *rain* is: 59.85385, therefore $\sum_{i=1}^{26}(x_i - \bar{x})^2 = (26-1)(59.85385) = 1496.346$. The mean of *rain* is computed $\bar{x} = 24.42308$.

We now run the regression of *wheat* on *rain* and obtain:

```
Call:
lm(formula = wheat ~ rain)

Residuals:
    Min      1Q  Median      3Q     Max
-131.57  -18.89   14.44   28.49   36.25

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  334.137     26.938  12.404 6.28e-12 ***
rain           1.149      1.053   1.091    0.286
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 40.74 on 24 degrees of freedom
Multiple R-Squared: 0.04722, Adjusted R-squared: 0.007518
F-statistic: 1.189 on 1 and 24 DF,  p-value: 0.2863
```

Therefore the estimate of σ^2 is $s_e^2 = 40.74^2 = 1659.748$.

The residuals, leverage values, and standardized residuals from this regression are listed below (from R):

Residuals:

```
          1           2           3           4           5           6           7 
-37.9216553 -30.2190978 -26.0703766 -22.5165403 -20.8139828  -9.1114253  -3.9627040 
          8           9          10          11          12          13          14 
  0.5911322  10.5911322 -13.1114253   9.4424110  18.2936898  20.9962473  27.8475260 
         15          16          17          18          19          20          21 
 28.6988048  32.5500835  34.4013623  36.2526410  18.9962473  22.8475260  25.6988048 
         22          23          24          25          26 
 30.5500835  31.4013623  33.2526410 -87.1114253 -131.5730626 
```

Leverage values:

```
         1          2          3          4          5          6          7          8 
0.14160134 0.11106542 0.12566508 0.08587585 0.06603264 0.05153579 0.05811592 0.04238530 
         9         10         11         12         13         14         15         16 
0.04238530 0.05153579 0.03981493 0.03858116 0.04012338 0.04289937 0.04701195 0.05246112 
        17         18         19         20         21         22         23         24 
0.05924688 0.06736923 0.04012338 0.04289937 0.04701195 0.05246112 0.05924688 0.06736923 
        25         26 
0.05153579 0.47564580 
```

Standardized residuals:

```
          1           2           3           4           5           6           7 
-1.00454535 -0.78663519 -0.68428208 -0.57799735 -0.52858656 -0.22961625 -0.10021199 
          8           9          10          11          12          13          14 
 0.01482573  0.26562795 -0.33041991  0.23650058  0.45790119  0.52596973  0.69860967 
         15          16          17          18          19          20          21 
 0.72151748  0.82069231  0.87049160  0.92132225  0.47586842  0.57317489  0.64609440 
         22          23          24          25          26 
 0.77026588  0.79457964  0.84508044 -2.19528759 -4.45945089 
```

Let's see how R computes these values. The leverage value for point i is equal to:

$$h_{ii} = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}.$$

Therefore, the leverage value of point 1 is:

$$h_{11} = \frac{1}{26} + \frac{(12 - 24.42308)^2}{1496.346} = 0.14160.$$

The standardized residual for point i is computed as follows:

$$e_i^* = \frac{e_i}{sd(e_i)} = \frac{e_i}{s_e\sqrt{1 - \frac{1}{n} - \frac{(x_i-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}} = \frac{e_i}{s_e\sqrt{1-h_{ii}}}.$$

Therefore the standardized residual for point 1 is equal to:

$$e_1^* = \frac{e_1}{s_e\sqrt{1-h_{11}}} = \frac{-37.9216553}{40.74\sqrt{1-0.14160134}} = -1.004666.$$

Any differences are due to rounding.