University of California, Los Angeles
Department of Statistics

Statistics 100C **Instructor: Nicolas Christou**

Practice problems - week 10

Answer the following questions:

a. Observations Y_1, Y_2, Y_3 and Y_4 are made of angles $\theta_1, \theta_2, \theta_3$, and θ_4 of a four-sided figure on the ground. If we assume that the observations are subject to independent normal errors with mean zero and variance σ^2 derive a test statistic for the hypothesis that the four-sided figure is a parallelogram with $\theta_1 = \theta_3$ and $\theta_2 = \theta_4$.

b. Given that $\mathbf{Y} = \boldsymbol{\theta} + \boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon} \sim N_4(\mathbf{0}, \sigma^2\mathbf{I})$ and $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$ show that the F statistic for testing the hypothesis $H_0 : \theta_1 = \theta_3$ is given by $\frac{2(Y_1 - Y_3)^2}{(Y_1+Y_2+Y_3+Y_4)^2}$.

c. Consider the centered and scaled multiple regression model $\mathbf{Y} = \gamma_0\mathbf{1} + \mathbf{Zs}\boldsymbol{\delta}_{(\mathbf{0})} + \boldsymbol{\epsilon}$. Suppose that we consider fitting a model in which the Y-data are centered and scaled as well as the x-data. (This means that the response variable is now $Y_{is} = \frac{Y_i - \bar{Y}}{\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$.) Obtain an expression for SSE for this new model and find its relationship to the SSE of the centered and scaled model where only the x-data are centered and scaled. Please show all the details.