University of California, Los Angeles
Department of Statistics

Statistics 100C **Instructor: Nicolas Christou**

Inverse of a partitioned matrix

If all inverses exist,

$$\begin{pmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A_{11}^{-1} + B_{12}B_{22}^{-1}B_{21}} & \mathbf{-B_{12}B_{22}^{-1}} \\ \mathbf{-B_{22}^{-1}B_{21}} & \mathbf{B_{22}^{-1}} \end{pmatrix} = \begin{pmatrix} \mathbf{C_{11}^{-1}} & \mathbf{-C_{11}^{-1}C_{12}} \\ \mathbf{-C_{21}C_{11}^{-1}} & \mathbf{A_{22}^{-1} + C_{21}C_{11}^{-1}C_{12}} \end{pmatrix}$$

where
$\mathbf{B_{22} = A_{22} - A_{21}A_{11}^{-1}A_{12}}$
$\mathbf{B_{12} = A_{11}^{-1}A_{12}}$
$\mathbf{B_{21} = A_{21}A_{11}^{-1}}$

and
$\mathbf{C_{11} = A_{11} - A_{12}A_{22}^{-1}A_{21}}$
$\mathbf{C_{12} = A_{12}A_{22}^{-1}}$
$\mathbf{C_{21} = A_{22}^{-1}A_{21}}$