Inverse of a partitioned matrix

If all inverses exist,

\[
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}^{-1} = \begin{pmatrix}
A_{11}^{-1} + B_{12}B_{22}^{-1}B_{21} & -B_{12}B_{22}^{-1} \\
-B_{21}B_{22}^{-1} & B_{22}^{-1}
\end{pmatrix} = \begin{pmatrix}
C_{11}^{-1} & -C_{11}^{-1}C_{12} \\
-C_{21}C_{11}^{-1} & A_{22}^{-1} + C_{21}C_{11}^{-1}C_{12}
\end{pmatrix}
\]

where

\[ B_{22} = A_{22} - A_{21}A_{11}^{-1}A_{12} \]
\[ B_{12} = A_{11}^{-1}A_{12} \]
\[ B_{21} = A_{21}A_{11}^{-1} \]

and

\[ C_{11} = A_{11} - A_{12}A_{22}^{-1}A_{21} \]
\[ C_{12} = A_{12}A_{22}^{-1} \]
\[ C_{21} = A_{22}^{-1}A_{21} \]

Use these results to show that the \( F \) test (\( F = \frac{MSR}{MSE} \)) for the overall significance of a multiple regression model is the same with the \( F \) test of the general linear hypothesis

\[ F_{m,n-k-1} = \frac{(C\hat{\beta} - \gamma)' [C(X'X)^{-1}C']^{-1} (C\hat{\beta} - \gamma)}{m s_e^2}, \]

where \( m \) is the rank of the matrix \( C \). For example, when we test the overall significance of the model (\( \beta_1 = \beta_2 = \ldots = \beta_k = 0 \)) we have \( m = k \). Reject \( H_0 \) if \( F_{m,n-k-1} > F_{1-\alpha;m,n-k-1} \).

Note:
In a multiple regression problem with \( k = 5 \) predictors the matrix \( C \) has the following form:

\[
C = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]