University of California, Los Angeles
Department of Statistics

Statistics 110A **Instructor: Nicolas Christou**

Poker combinations

A Poker hand consists of five cards. Players try for combinations of two or more cards of a kind, five-card sequences, or five cards of the same suit. Poker is played with a standard 52-card deck in which all suits are of equal value, the cards ranking from the ace high, downward through king, queen, jack, and the numbered cards 10 to the deuce. The ace may also be considered low to form a straight (sequence) ace through five as well as high with king-queen-jack-10.

Here is the traditional ranking of hands with some examples:

1. Royal Flush (ten-jack-queen-king-ace all of the same suit): $10\clubsuit, J\clubsuit, Q\clubsuit, K\clubsuit, A\clubsuit$.
2. Straight Flush (five cards of the same suit in sequence other than royal flush): $2\spadesuit, 3\spadesuit, 4\spadesuit, 5\spadesuit, 6\spadesuit$.
3. Four of a kind, plus any fifth card: $5\clubsuit, 5\spadesuit, 5\diamondsuit, 5\heartsuit, 4\clubsuit$.
4. Full house (three of a kind plus a pair): $5\clubsuit, 5\spadesuit, 5\diamondsuit, 4\heartsuit, 4\clubsuit$.
5. Flush (any five cards of the same suit other than straight flush): $3\clubsuit, 6\clubsuit, 8\clubsuit, 10\clubsuit, J\clubsuit$.
6. Straight (five cards in sequence not all of the same suit): $2\spadesuit, 3\heartsuit, 4\spadesuit, 5\clubsuit, 6\diamondsuit$.
7. Three of a kind plus no pair: $5\clubsuit, 5\spadesuit, 5\diamondsuit, 4\heartsuit, 6\clubsuit$.
8. Two pairs plus any fifth card: $5\clubsuit, 5\spadesuit, 4\diamondsuit, 4\heartsuit, 9\clubsuit$.
9. One pair plus three other cards: $5\clubsuit, 5\spadesuit, 4\diamondsuit, 6\heartsuit, 10\clubsuit$.
10. Nothing

Rank	Poker Hand	# of Combinations
1.	Royal Flush	4
2.	Other Straight Flush	36
3.	Four of a kind	624
4.	Full House	3744
5.	Flush	5108
6.	Straight	10200
7.	Three of a kind	54912
8.	Two Pairs	123552
9.	One Pair	1098240
10.	Nothing	1302540
	Total	2598960

A standard 52-card deck consists of the following 13 denominations (4 cards in eac denomination), and 4 suits (13 cards in each suit):

$\clubsuit$	$\spadesuit$	$\diamondsuit$	$\heartsuit$
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
J	J	J	J
Q	Q	Q	Q
K	K	K	K
A	A	A	A

There are $\binom{52}{5} = \frac{52!}{(52-5)!5!} = 2598960$ different ways to choose 5 cards from the available 52 cards.

Let's compute the number of combinations of the following poker hand: four of kind plus any fifth card: We need 2 different denominations (for example 4 aces plus an eight). There are $\binom{13}{2}$ different ways to choose 2 denominations from the 13 available denominations. Now, there are $\binom{4}{4}$ ways to choose the 4 aces from the 4 aces, and $\binom{4}{1}$ different ways to choose one eight from the 4 eights. Also, the opposite could have occured (that is, 4 eights and 1 ace). Therefore the number of combinations of a four of a kind plus any fifth card is:

$$\binom{13}{2}\binom{4}{4}\binom{4}{1}2 = 624.$$

Similarly, we can find the number of combinations of a pair plus 3 other cards (for example 2 aces plus 1 two, 1 seven, 1 eight). We need 4 denominations. There are $\binom{13}{4}$ different ways to choose 4 denominations from the available 13 denominations. Now, there are $\binom{4}{2}$ different ways to choose a pair from one of the 4 denominations, and $\binom{4}{1}$ different ways to choose one card from each of the other 3 denominations. Also, we multiply by 4 because the pair could have come from the 2s or the 7s or the 8s. Therefore the number of combinations of a pair plus 3 other cards is:

$$\binom{13}{4}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}4 = 1098240.$$

For the examples above, to compute the probability that a player receives a four of kind plus any fifth card or a pair plus 3 other cards we simply divide the number of combinations that this particular poker hands occurs over the total number of ways in which 5 cards can be selected from the available 52 cards.

$$P(\text{four of a kind plus a fifth card}) = \frac{\binom{13}{2}\binom{4}{4}\binom{4}{1}2}{\binom{52}{5}} = \frac{624}{2598960} = 0.00024.$$