University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Exam 1
23 October 2013

Name: ____________________________

Problem 1 (20 points)
Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ using a sample of size $n = 10$, which yielded

$$\sum_{i=1}^{n} y_i = 63.68, \sum_{i=1}^{n} x_i = 190, \sum_{i=1}^{n} y_i^2 = 446.33, \sum_{i=1}^{n} x_i^2 = 3940, \sum_{i=1}^{n} x_i y_i = 1100.$$

a. Find the least-squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.

b. Find s_e^2, the estimate of the variance σ^2.

c. Find the coefficient of determination R^2 and the correlation coefficient r.

d. Suppose that $\sum_{i=1}^{n} x_i = 0$. What are the least squares estimates of β_0 and β_1?

Problem 2 (20 points)
Using the `stockPortfolio` package we obtain data for three stocks (Apple Inc., Exxon Mobil Corporation, Facebook. Inc.), and the market index $S\&P500$ (`GSPC`).

```
> library(stockPortfolio)
> ticker <- c("AAPL", "XOM", "FB", "^GSPC")
> gr <- getReturns(ticker, start="2011-05-31", end="2013-09-30")

> colMeans(as.data.frame(gr$R))
   AAPL      XOM       FB    ^GSPC
-0.0072   0.0083   0.0510   0.0160

> cov(gr$R)
         AAPL     XOM      FB    ^GSPC
AAPL  0.00623 0.00009 0.00185 0.00032
XOM   0.00009 0.00123 0.00014 0.00059
FB    0.00185 0.00014 0.03927 0.00116
^GSPC 0.00032 0.00059 0.00116 0.00056
```

Answer the following questions:

a. Suppose you want to run the regression of the returns of `AAPL` on the index $S\&P500$. Find the beta of the stock.

b. Consider the stocks `AAPL` and `XOM`. Find the composition of the minimum risk portfolio using these two stocks.

c. What is the correlation coefficient between the stocks `AAPL` and `FB`.

d. Consider the regression of part (a). Compute the estimate of α of `AAPL`.

Problem 3 (20 points)
Answer the following questions:

a. Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, with $i = 1, 2, \ldots, n$. Show that $\sum_{i=1}^{n} e_i x_i = 0$.

b. Consider the model of part (a). Suppose you want to run the regression of the residuals on the predictor variable x. What are the estimates of the values of the intercept and slope of this regression?

c. Suppose you are dealing with the following model: $y_i = \mu + \epsilon_i$, with $i = 1, 2, \ldots, n$. Use the method of least squares to find the estimate of the unknown parameter μ.

d. Consider the variables `lead` and `zinc` of the data soil discussed in class and in the lab. The following results are obtained using these two variables:

```
> nrow(a)
[1] 155

> cov(a[,3:4])
         lead      zinc
lead 12392.15  39011.24
zinc 39011.24 134743.17

> colMeans(a[,3:4])
    lead     zinc
153.3613 469.7161

> a[153:155,]
         x      y lead zinc
153 178875 330311  119  342
154 179466 330381   51  162
155 180627 330190  124  375
```

Find the leverage value that corresponds to row 155 of the data set. Is this a high influential point in the regression of `lead` on `zinc`.

Problem 4 (20 points)

Suppose two dice are in a box. One is a fair die, so that the probability of any face is $\frac{1}{6}$. The other die is not fair, and the probability of an i is $\frac{i}{21}$, for $i = 1, 2, 3, 4, 5, 6$. Suppose a die is chosen at random form the box and rolled twice. Let A be the event that the first roll gives a 6 and B be the event that the second roll gives a 6. Answer the following questions:

a. Find $P(A), P(B)$, and $P(A \cap B)$.

b. Are A and B independent?

c. What is the conditional probability of a 6 on the second roll given a 6 on the first roll?

d. Given a six on the first and second rolls what is the probability that the fair coin was chosen?

Problem 5 (20 points)
Answer the following questions:

a. Two physicians A and B are on call during nonworking hours. Physician A is within the distance over which he can hear his beeper 85% of the time, while for physician B the probability is 70%. The two physicians are independent of each other. Answer the following questions: Find the probability that at least one of the two physicians will hear his/her beeper.

b. Refer to question (a). The hospital is thinking of adding a third physician, C. The probability that she will hear her beeper is 80% of the time. If they do this, what is the probability that at least one of the three physicians will hear his/her beeper. Assume that the three physicians are independent.

c. Suppose A, B, and C are three events such that
A, B are disjoint,
A, C are independent, and
B, C are independent.
Suppose also that $4P(A) = 2P(B) = P(C)$, and $P(A \cup B \cup C) = 5P(A)$. Find $P(A)$.

d. Suppose 5 cards are selected from an ordinary deck of 52 playing cards without replacement. Find the conditional probability that all the 5 cards are clubs, given that there are at least 4 clubs.