University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Exam 1
29 October 2014

Name: ______________________________

UCLA ID: __________________________

Section: ______

Problem 1 (20 points)
Answer the following questions:

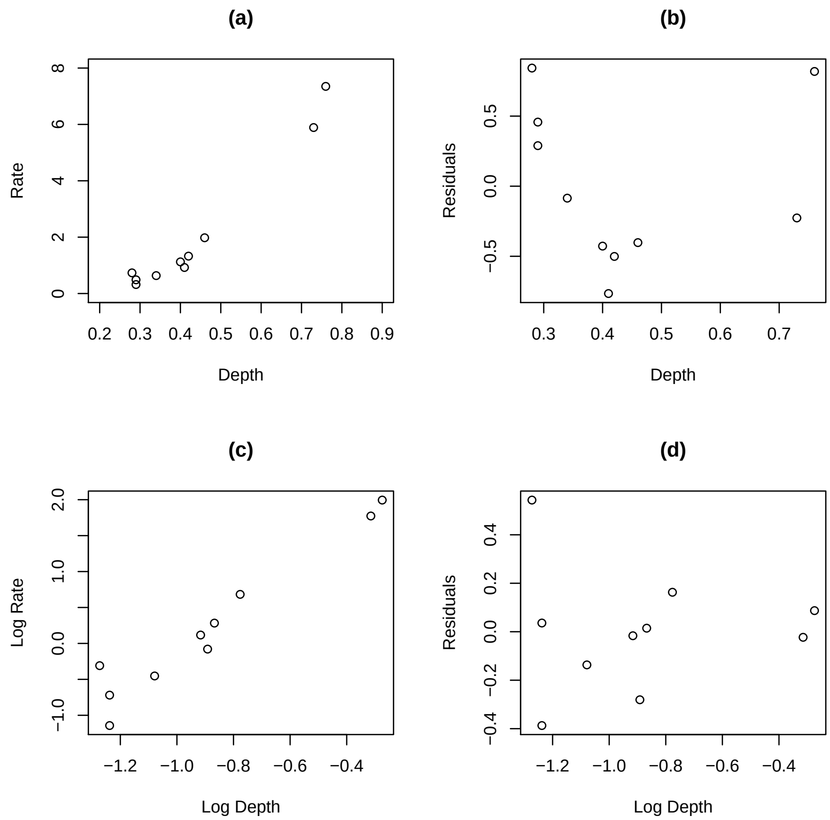
a. Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Using data on y and x we find the least squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ and fit the best line $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$. Is it possible that $var(y) < s_e^2$? Note: $var(y)$ is the variance of the y values and s_e^2 is the variability around the fitted line. If this is possible can you draw (approximately) the corresponding scatterplot of y on x?

b. Consider the simple regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. If $\beta_0 = 4$ what is the least squares estimate of β_1? Note: Use the procedure of least squares we discussed in class, but here the intercept is known!

c. Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. One of the goals of regression is to use the fitted regression line to make predictions for y when a new value of x is given. If a new value of x is below $\bar{x}$ is it true that the predicted value of y will be always below $\bar{y}$? Please explain your answer!

Problem 2 (20 points)
The data in the table below shows the depth of a stream and the rate of its flow.

Flow Rate	Depth
0.636	0.34
0.319	0.29
0.734	0.28
1.327	0.42
0.487	0.29
0.924	0.41
7.350	0.76
5.890	0.73
1.979	0.46
1.124	0.40



Plot (a) shows the scatterplot of `Rate` against `Depth`.
Plot (b) shows the scatterplot of `Residuals` against `Depth` from the simple regression of `Rate` on `Depth`.
Plot (c) shows the scatterplot of `Log(Rate)` against `Log(Depth)`.
Plot (d) shows the scatterplot of `Residuals` against `Log(Depth)` from the simple regression of `Log(Rate)` on `Log(Depth)`.

Answer the following questions:

a. Write the regression model equation based on plot (a).

b. Write the fitted line equation based on plot (c).

c. Explain why it was necessary to transform the data using logarithms.

Problem 3 (20 points)

Part A:

Data on `cadmium` (x) and `cobalt` (y) of soil at a certain area gave the following information:

$\sum_{i=1}^{6} x_i = 9.545$,

$\sum_{i=1}^{6} y_i = 61.668$,

$\sum_{i=1}^{6} x_i^2 = 15.78468$,

$\sum_{i=1}^{6} y_i^2 = 719.9573$,

$\sum_{i=1}^{6} x_i y_i = 96.43722$.

It was discovered that the last two x values were interchanged. The incorrect table is shown below:

x	y
.	.
.	.
.	.
.	.
1.145	16.320
1.565	3.508

Find the corrected $\hat{\beta}_1$ from the regression of y on x.

Part B:

Data on `cadmium` (x) and `cobalt` (y) of soil at a certain area gave the following information:

$\sum_{i=1}^{6} x_i = 11.908$,

$\sum_{i=1}^{6} y_i = 59.305$,

$\sum_{i=1}^{6} x_i^2 = 26.77971$,

$\sum_{i=1}^{6} y_i^2 = 708.9622$,

$\sum_{i=1}^{6} x_i y_i = 101.8183$.

It was discovered that a pair of x and y was interchanged. The incorrect table is shown below:

x	y
.	.
.	.
.	.
.	.
.	.
3.508	1.145

Find the corrected $\hat{\beta}_1$ from the regression of y on x.

Problem 4 (20 points)
Answer the following questions:

a. Plot the ecdf of the following numbers: 1, 13, 9, 8, 10, 8.

b. Consider the body fat data (see handout "Introduction to R"). We use here the variables y, x_6, x_9, x_{10}. The variance covariance matrix and the sample means of the four variables are shown next:

```
#Variance covariance matrix:
          y     x6     x9    x10
y    70.036  9.980 37.483 24.587
x6    9.980  5.909 12.799  8.879
x9   37.483 12.799 51.324 33.715
x10  24.587  8.879 33.715 27.562

#Sample means:
     y     x6     x9    x10
19.151 37.992 99.905 59.406
```

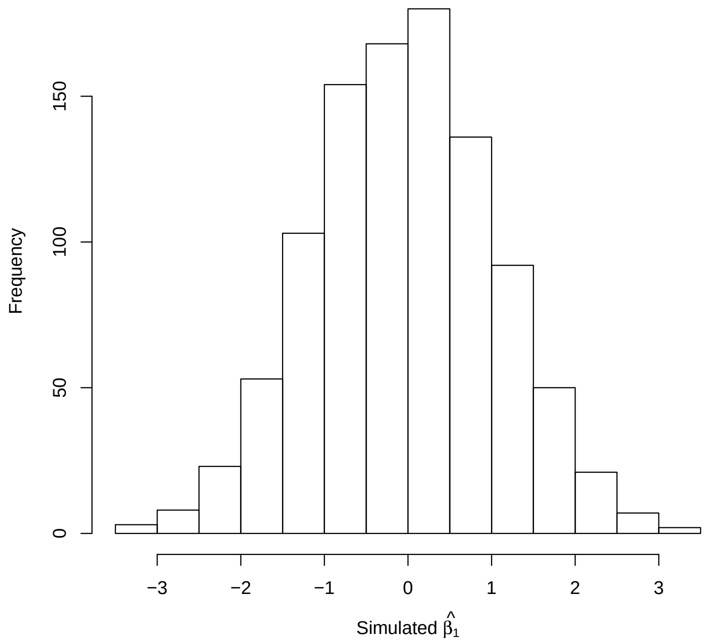
Find the fitted line of the regression of y on x_9. Show all your work.

c. Refer to part (b). Compute the correlation coefficient between y and x_{10}.

d. Refer to part (b). Consider the regression of y on x_6. Compute the value of the regression sum of squares.

Problem 5 (20 points)
Normal temperature (degrees Fahrenheit) readings (x) and heart rates (beats per minute) of 65 males and 65 females were used in this problem.

a. Consider the data for males: The OLS estimate of β_1 is $\hat{\beta}_1 = 1.65$. A permutation test was run to test the hypothesis $H_0 : \beta_1 = 0$ against the alternative $H_a : \beta_1 \neq 0$ of the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Under the null hypothesis the permutation test produced the following histogram. What is your conclusion?



a. Consider the data for females: The OLS estimate of β_1 is $\hat{\beta}_1 = 3.13$. A permutation test was run to test the hypothesis $H_0 : \beta_1 = 0$ against the alternative $H_a : \beta_1 \neq 0$ of the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Under the null hypothesis the permutation test produced the following histogram. What is your conclusion?