

University of California, Los Angeles
Department of Statistics

Statistics 13

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Exam 2
14 May 2009

Name: _____

Problem 1 (20 points)

Answer the following questions:

- a. You are about to take a random sample in order to use \bar{X} to estimate μ . You want the following to hold:

$$P(\mu - 5 \leq \bar{X} \leq \mu + 5) \geq 0.80$$

Suppose that the population standard deviation is equal to $\sigma = 40$. Find the smallest sample size n which will allow you to satisfy the condition above.

- b. A population has mean $\mu = 7$ and standard deviation $\sigma = 2.42$. What is the probability that among 5 samples (each of size $n = 50$) three sample means will be larger than 7.5?
- c. Consider the distribution of the sample proportion \hat{p} . Suppose candidate A receives 60% of the vote, and a sample of 500 voters is selected. Find the c such that $P(\hat{p} > c) = 0.80$.

Problem 2 (30 points)

Three independent random samples of size $n_1 = 16$, $n_2 = 9$, and $n_3 = 4$ are selected from the normal distribution $N(16, 5.2)$ ($\mu = 16, \sigma = 5.2$).

- a. The graph on the next page shows the distribution of the sample means $\bar{X}_1, \bar{X}_2, \bar{X}_3$ for these three samples. Describe the distribution of $\bar{X}_1, \bar{X}_2, \bar{X}_3$ and match them to the letters (a), (b), (c) on the graph.
- b. What is the distribution of $2\bar{X}_1 - \bar{X}_2 - \frac{1}{2}\bar{X}_3$?
- c. Suppose that the population mean μ is unknown. The first sample ($n_1 = 16$) gave $\bar{X}_1 = 18.56$ and $s = 6.19$. Construct a 90% confidence interval for the population mean μ .
- d. You want to reduce the margin of error of question (c), so you decided to combine the values of the three samples to obtain a larger sample of size 29. Do you think that the margin of error will (i) decrease, (ii) increase, (iii) we don't know. Please explain your answer.
- e. Find c such that $P(\bar{X}_1 > c) = 0.22$.

Problem 3 (25 points)

The top 20 most popular baby names for the year 2008 are shown on the next page. For more information visit <http://www.ssa.gov/OACT/babynames>.

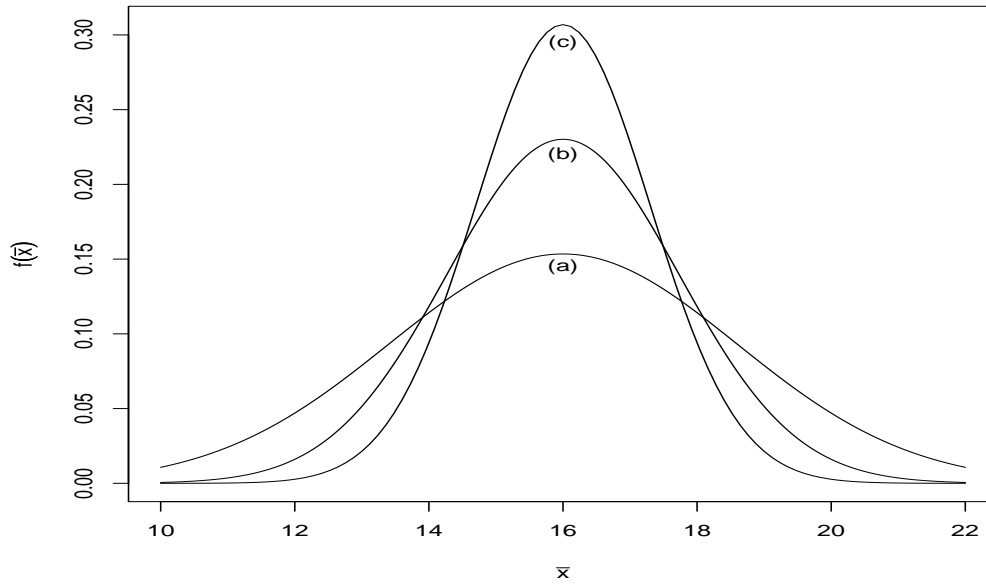
- a. A toy company randomly selects 20000 babies and will give free toys to the babies named "Alyssa". Assume that these 20000 babies are selected from a very large population. Write the expression that computes the exact probability that they will find more than 100 babies named "Alyssa".
- b. Approximate the probability of part (a).
- c. What is the probability that the sample proportion of the names "Alyssa" will exceed 0.0053?
- d. Which one is larger $P(\hat{p} > 0.0053, \text{ when } n = 20000)$ or $P(\hat{p} > 0.0053, \text{ when } n = 40000)$? To answer this question use only graphs. No calculations!

Problem 4 (25 points)

Answer the following questions:

- a. The distribution of the points scored by the Los Angeles Lakers during the regular NBA season is approximately normal with mean 110 points and standard deviation 5 points. What is the probability that in 6 playoff games the total number of points scored by the Los Angeles Lakers will be between 647 and 680 points?
- b. Suppose $X \sim N(100, 3)$. After observing a value of X a rectangle is constructed with length $L = X$ and width $W = 3X$. Let A denote the area of the resulting rectangle. Find the expected value of the area, $E(A)$.
- c. The lifetime of an electronic component follows the exponential distribution with mean $\mu = 10$ months and standard deviation $\sigma = 10$ month. What can we say about the distribution of the sample mean of 5 such electronic components?
- d. Redwood trees (*sequoia sempervirens*), native to the northwest coastal California, are the tallest trees on Earth. The height of one of the tallest redwood trees is 106.5 meters. Assuming that this height is the 97_{th} percentile of the distribution and that the standard deviation is $\sigma = 3$ meters, find the mean of the population.

Problem 2(a)



Problem 3

Popular Baby Names

Popular Names by Birth Year
May 9, 2009

Popularity in 2008

Rank	Male name	Percent of total males	Female name	Percent of total females
1	Jacob	1.0355%	Emma	0.9043%
2	Michael	0.9437%	Isabella	0.8941%
3	Ethan	0.9301%	Emily	0.8377%
4	Joshua	0.8799%	Madison	0.8199%
5	Daniel	0.8702%	Ava	0.8198%
6	Alexander	0.8566%	Olivia	0.8196%
7	Anthony	0.8442%	Sophia	0.7729%
8	William	0.8438%	Abigail	0.7250%
9	Christopher	0.8268%	Elizabeth	0.5748%
10	Matthew	0.8061%	Chloe	0.5692%
11	Jayden	0.7877%	Samantha	0.5384%
12	Andrew	0.7694%	Addison	0.5188%
13	Joseph	0.7570%	Natalie	0.4905%
14	David	0.7467%	Mia	0.4898%
15	Noah	0.7262%	Alexis	0.4671%
16	Aiden	0.7165%	Alyssa	0.4640%
17	James	0.6952%	Hannah	0.4582%
18	Ryan	0.6754%	Ashley	0.4530%
19	Logan	0.6372%	Ella	0.4497%
20	John	0.6102%	Sarah	0.4332%

Note: Rank 1 is the most popular, rank 2 is the next most popular, and so forth.
All names are from Social Security card applications for births that occurred in the United States.