University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Exam 2
16 November 2012

Name: ____________________________

Problem 1 (25 points)
Answer the following questions:

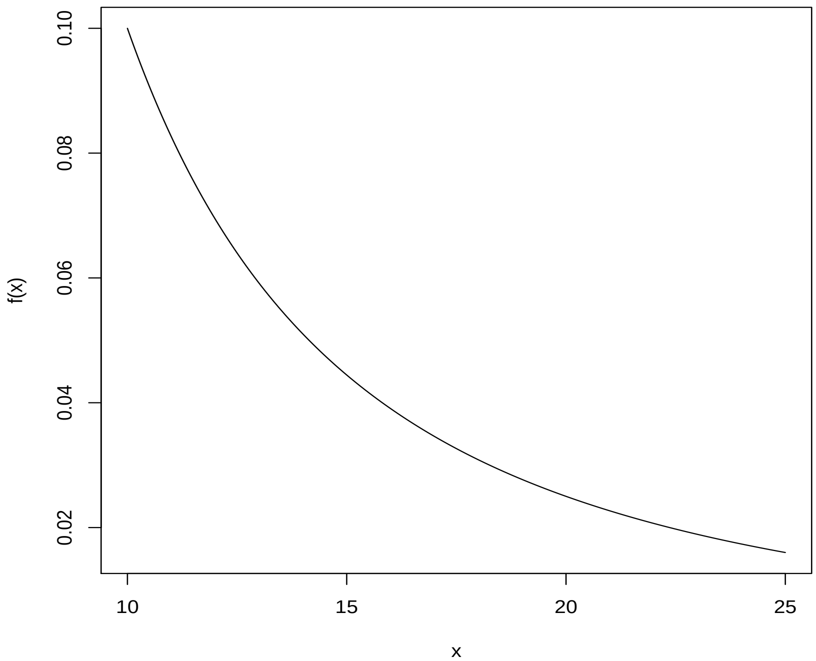
a. Player A has probability 0.4 of winning a game, while player B has probability 0.8. Player A will play the game until she wins 5 times. Player B will play the game until she wins 12 times. Which player would you expect to play the game fewer times?

b. Many people cancel their hotel reservations at the last minute. Therefore, most hotels overbook when possible. A particular hotel has 90 rooms and it is known that about 20% of the people with reservations cancel them. On a particular night there are 100 reservations. What is the approximate probability that the people with reservations will fill at most $\frac{5}{6}$ of the hotel?

c. Suppose the number of particles in a sample of water follows the Poisson distribution with $\lambda = 100$ particles per cubic inch. Find the probability of at least two particles in a volume of 0.02 cubic inch.

d. Let $X \sim N(100, 10)$. Find the interval that covers the middle 60% of this distribution.

Problem 2 (25 points)
Suppose the lifetime X (in months) of an electronic component is a continuous random variable with probability density function (pdf) $f(x) = \frac{10}{x^2}, x > 10$. It is also given that the cumulative distribution function (cdf) is $F(x) = P(X \leq x) = 1 - \frac{10}{x}$. The density is shown below:



Answer the following questions:

a. Use the cdf to compute $P(X > 20)$.

b. Use the cdf to find the 75_{th} percentile of the distribution of X.

c. Compute the probability that among 6 such electronic components, at least two will survive more than 15 months.

d. Compute $P(X > 30|X > 20)$.

Problem 3 (25 points)
Answer the following questions:

a. Suppose the lifetime of an electronic component follows the exponential distribution with parameter λ. Show that $P(X > s+t|X > t) = P(X > s)$, with $s > 0, t > 0$. Note: This is called the memoryless property of the exponential distribution.

b. Suppose that you bet \$5 on each of a sequence of 50 independent fair games. Use the central limit theorem to approximate the probability that you will lose more than \$75.

c. Suppose that a certain auto part has mean μ grams and standard deviation $\sigma = 5$ grams. Let $\bar{X}$ be the sample mean of n such auto parts. How large should n be so that $P(|\bar{X} - \mu| < 1) = 0.95$?

d. Suppose that the number of insurance claims, Y, filed in a given year follows the Poisson probability distribution with $E(Y) = 10000$. Use the normal approximation to Poisson to compute $P(Y > 10200)$.

Problem 4 (25 points)
Answer the following questions:

a. Suppose that on a certain examination in advanced mathematics, students from university A achieve scores that follow $N(625, 10)$, and students form university B achieve scores that follow $N(600, \sqrt{150})$. If two students from university A and three students from university B take this examination, what is the probability that the average score of the two students from university A will be greater than the average score of the three students from university B? All the scores are independent of each other.

b. Suppose that people attending a party pour drinks from a bottle containing 63 ounces of orange juice. Suppose also that the expected size (mean) of each drink is 2 ounces, that the standard deviation of each drink is 0.5 ounce, and that all drinks are poured independently. Find the probability that the bottle will not be empty after 36 drinks have been poured.

c. A city has 100000 voting people and suppose that 60000 of these voters watch a particular program. We sample five people without replacement from the entire population of voters. Approximate the probability that we find exactly two watching the program.

d. Let the number of cars X that arrive at a parking lot follow the Poisson distribution with parameter $\lambda = 60$ cars per hour. You are at the parking lot waiting for the next car to arrive. Let's denote with Y the time that you have to wait until the next car arrives. We want to know the distribution of Y. Here are some steps to help you. Please answer as many questions as you can from the list below:

    1. Is Y continuous or discrete?
    2. Let's denote with $P(Y > 2)$ the probability that the next car will arrive after time $y = 2$ minutes.
    3. If the next car arrives after two minutes it means that during the 2-minute interval there are no arrivals. What is the value of the parameter λ for this 2-minute interval?
    4. Therefore, $P(Y > 2 \; minutes) = P(X = 0 \; cars)$. You should be able to compute$P(X = 0)$ for the 2-minute interval. After you compute this probability (which is also equal to $P(Y > 2)$), try to compare it with a function that reminds you a known distribution.