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Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Exam 2 - practice problems

Problem 1

Answer the following questions:

a. A die is rolled until we observe number 5 for the first time. What is the probability that we observe number 5 for the first time after the 8_{th} trial?
$P(X > 8) = (\frac{5}{6})^8 = 0.2326$.

b. Which ones of the following are random and which ones are not random? $\bar{X}, \mu, T$.
$\bar{X}, T$ are random, μ is not random.

c. In a basket there are 2 oranges and 1 apple. Two fruits are selected without replacement. Construct the probability distribution of X, where X represents the number of oranges among the 2 fruits selected.

X	$P(X)$
1	$\frac{\binom{2}{1}\binom{1}{1}}{\binom{3}{2}} = \frac{2}{3}$
2	$\frac{\binom{2}{2}\binom{1}{0}}{\binom{3}{2}} = \frac{1}{3}$

d. Let $X \sim N(10, 2)$. Find $P(X > 15/X > 13)$.
$P(X > 15/X > 13) = \frac{P(X>15 \cap X>13)}{P(X>13)} = \frac{P(X>15)}{P(X>13)} = \frac{P(X>\frac{15-10}{2})}{P(Z>\frac{13-10}{2})} = \frac{P(Z>2.5)}{P(Z>1.5)} = \frac{1-0.9938}{1-0.9332} =$ 0.0928.

Problem 2

A selective college would like to have an entering class of 1200 students. Because not all students who are offered admission accept, the college admits 1500 students. Past experience shows that 70% of the students admitted will accept. Assuming that students make their decisions independently, the number who accept X follows the binomial distribution with $n = 1500$ and $p = 0.70$.

a. Write an expression for the exact probability that at least 1000 students accept.
$P(X \geq 1000) = \sum_{x=1000}^{1500} \binom{1500}{x} 0.70^x (1 - 0.70)^{1500-x}$.

b. Approximate the above probability using the normal distribution.
$P(X \geq 1000) = P(Z > \frac{999.5-1500(0.70)}{\sqrt{1500(0.70)(1-0.70)}}) = P(Z > -2.85) = 1 - 0.0022 = 0.9978$.

Problem 3

A bag of cookies is underweight if it weighs less than 500 grams. The filling process dispenses cookies with weight that follows the normal distribution with mean 510 grams and standard deviation 4 grams.

a. What is the probability that a randomly selected bag is underweight?
   $P(X < 500) = P(Z < \frac{500-510}{4}) = P(Z < -2.5) = 0.0062.$

b. If you randomly select 5 bags, what is the probability that exactly 2 of them will be underweight?
   $P(Y = 2) = \binom{5}{2} 0.0062^2 (1 - 0.0062)^3 = 0.0004.$

Problem 4

An insurance company has 10000 automobile policyholders. The mean yearly claim per policyholder is \$240 with a standard deviation of \$800.

a. Describe the distribution of the total yearly claim?
   $T \sim N(10000(240), 800\sqrt{10000})$ or $T \sim N(2400000, 80000)$.

b. Find the probability that the total yearly claim exceeds \$2.7 million.
   $P(T > 2700000) = p(Z > \frac{2700000-2400000}{80000}) = P(Z > 3.75) = 1 - 0.9999 = 0.0001.$

c. The probability that the total yearly claim exceeds b is 2.5%. Find b.
   $1.96 = \frac{b-2400000}{80000} \Rightarrow b = 2556800.$