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Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Problem 1
Answer the following questions:

a. The effectiveness of a new drug that lowers the cholesterol level will be tested against a placebo. The placebo will be administered to $n_1 = 30$ patients while the drug will be administered to $n_2 = 30$ patients. The hypothesis we want to test is
$H_0 : \mu_1 - \mu_2 = 10$
$H_a : \mu_1 - \mu_2 > 10$
Assume that $\sigma_1^2 = 35, \sigma_2^2 = 25$. Find the probability of detecting a shift from $\mu_1 - \mu_2 = 10$ to $\mu_1 - \mu_2 = 13$ if we are willing to accept a Type I error of $\alpha = 0.05$.

b. Find the sample size n needed for each group in order to detect with probability 99% a shift from $\mu_1 - \mu_2 = 10$ to $\mu_1 - \mu_2 = 13$ if $\alpha = 0.05$.

Problem 2
The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of a certain type of steel, the Rockwell index gave a sample mean of $\bar{x} = 62$ with sample standard deviation $s = 8$. The manufacturer claims that this steel has a mean hardness index of 64.

a. Using $\alpha = 0.05$ level of significance test whether the mean Rockwell hardness index is less than 64. Please approximate the p-value in order to test the hypothesis.

b. Based on your answer to part (a), is it still possible that the null hypothesis is true?

The steel is sufficiently hard for a certain use so long as the mean Rockwell hardness measure does not drop below 60.

c. Suppose that the true mean is 60. If $\alpha = 0.05$ what is the probability of detecting the shift from 64 to 60? For this question assume that $\sigma = 8$.

Suppose that we want the probability of detecting the shift from 64 to 60 to be 99%. Assume that we are willing to take a risk of commiting a Type I error $\alpha = 0.05$. For this question assume that $\sigma = 8$

d. What sample size should we select?

Problem 3
Let X be a uniform random variable on $(0, \theta)$. You have exactly one observation from this distribution and you want to test the null hypothesis $H_0 : \theta = 10$ against the alternative $H_a : \theta > 10$, and you want to use significance level $\alpha = 0.10$. Two testing procedures are being considered:
Procedure G rejects H_0 if and only if $X \geq 9$.
Procedure K rejects H_0 if either $X \geq 9.5$ or if $X \leq 0.5$.

a. Confirm that Procedure G has a Type I error probability of 0.10.

b. Confirm that Procedure K has a Type I error probability of 0.10.

c. Find the power of Procedure G when $\theta = 12$.

d. Find the power of Procedure K when $\theta = 12$.

Problem 4

Results of a laboratory analysis of calories content of major hot dog brands are given below. Researchers for *Consumer Reports* analyzed three types of hot dog: beef, poultry, and meat (mostly pork and beef, but up to 15% poultry meat). The data are summarized in the table below:

Type	n_i	$\bar{y}_i$	s_i^2
Beef	20	156.85	512.66
Poultry	17	118.76	508.57
Meat	17	158.71	636.85

Reference: Moore, David S., and George P. McCabe (1989), *Introduction to the Practice of Statistics.* Original source: *Consumer Reports*, June 1986, pp. 366-367.

Answer the following questions:

a. Construct the ANOVA table and test the hypothesis that the mean calories content for the three types of hot dogs are equal against the alternative that at least two are not equal. Use $\alpha = 0.05$. What do you conclude?

Complete the ANOVA table:

Source	d.f.	SS	MS	F
Between				
Within				
Total				

b. Construct a Bonferroni confidence interval for the difference between the means of beef and poultry. You will need this value: $t_{0.00833;51} = 2.476$. What do you conclude?

Problem 5

Making use of the fact that the χ^2 distribution can be approximated with the normal distribution when the number of degrees of freedom is large, show that for large samples from normal populations

$$s^2 \geq \sigma_0^2 \left[1 + z_\alpha \sqrt{\frac{2}{n-1}}\right]$$

is an approximate critical region of size α for testing:

$H_0 : \sigma^2 = \sigma_0^2$

$H_a : \sigma^2 > \sigma_0^2$