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Homework 4

EXERCISE 1
Two balls are chosen randomly without replacement from and urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. We neither win nor we lose any money for selecting an orange ball. Let X denote our winnings.

a. What is the expected value of X?

b. What is the variance of X?

c. Given that our winnings are negative, what is the probability that we lost exactly \$2?

EXERCISE 2
Only 4% of people have type AB blood.

a. On average, how many donors must be checked to find someone with type AB blood?

b. What is the probability that there is a type AB blood donor among the first 5 people checked?

c. What is the probability that the first type AB donor will be found among the first 6 people?

d. What is the probability that the first type AB donor will be found on or after the $10th$ person checked?

e. What is the probability that the first type AB donor will be found on or before the $5th$ person checked?

EXERCISE 3
Two players independently play roulette. They bet on a single number (it pays $35:1$).

a. Find the expected value of the sum of their winnings.

b. Find the variance of the sum of their winnings.

EXERCISE 4
Answer the following questions:

a. Cards are selected with replacement until the first ace is found. What is the probability that the first ace will be found before the 3_{rd} trial?

b. Ten cards are selected without replacement from a standard 52 card deck that contains 13 clubs and 39 other cards. What is the probability that 4 of them are clubs?

c. Ten cards are selected with replacement. What is the probability that 4 of them are clubs?

d. Each of 2 players, independently, play a game 5 times, with each game having probability of success 0.40. What is the standard deviation of the sum of the games they win?

EXERCISE 5
To determine whether or not they have a certain disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people. If the test is positive each of the 10 people will also be individually tested. Suppose the probability that a person has the disease is 0.10 for all people independently from each other. Compute the expected number of tests necessary for each group.

EXERCISE 6
In exercise 5, the 100 people were placed in groups of 10. Try different group sizes in order to minimize the expected number of tests? For example, you can try 2, 4, 5, 20 etc? What group size gives the least expected number of tests?

EXERCISE 7
A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 components are nondefective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject?

EXERCISE 8
New York Lotto is played as follows: Out of 59 numbers 6 are chosen at random without replacement. Then from the remaining 53 numbers 1 is chosen. This last number is called "the bonus number". You, the player, select 6 numbers. To win the first prize you must match your 6 numbers with the State's 6 numbers. If you match only 5 numbers and your 6_{th} number matches the bonus number then you win the second prize.

a. What is the probability of winning the first prize?

b. What is the probability of winning the second prize?

c. What is the probability of winning a prize (either the first or the second)?

Note: Check your answers to (a, b) at *http://nylottery.ny.gov/wps/portal* .

EXERCISE 9
Consider the simple regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, for $i = 1, 2, \ldots, n$. Assume that $E(\epsilon_i) = 0$, $var(\epsilon_i) = \sigma^2$, and $cov(\epsilon_i, \epsilon_j) = 0$, for $i \neq j$. We know enough about expectation and variance of random variables to answer the following questions.

a. Find $E(\hat{\beta}_1)$.

b. Find $E(\hat{\beta}_0)$.

EXERCISE 10
Refer to the model of exercise 9.

a. Show that $cov(\bar{Y}, \hat{\beta}_1) = 0$.

b. Find $var(\hat{\beta}_1)$.

c. Find $var(\hat{\beta}_0)$.