University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1
Suppose that the cholesterol level in the US adult women population follows the normal distribution with mean $\mu = 188mg/dL$, and standard deviation $\sigma = 24mg/dL$.

a. Sketch and label the normal curve.

b. Find the 25th and 75th percentile of this distribution, and calculate the interquartile range.

c. What proportion of women have cholesterol level above $220mg/dL$?

d. What proportion of women have cholesterol level between 150 and $170mg/dL$?

EXERCISE 2
Suppose that the height (X) in inches, of a 25-year-old man is a normal random variable with mean $\mu = 70$ inches. If $P(X > 79) = 0.025$ what is the standard deviation of this random normal variable?

EXERCISE 3
Suppose that the weight (X) in pounds, of a 40-year-old man is a normal random variable with standard deviation $\sigma = 20$ pounds. If 5% of this population is heavier than 214 pounds what is the mean μ of this distribution?

EXERCISE 4
The length of the tails of a certain breed of dogs follows the normal distribution with mean μ and standard deviation σ. It is known that 5% of the tails is longer than 12 inches. It is also known that 2.5% of the tails is shorter than 7 inches.

a. Find μ and σ.

b. Suppose that a dog is randomly selected. What is the probability that the length of its tail will be longer than 11 inches?

EXERCISE 5
An architect is designing the interior doors in an office building. She desires to make the heights of the doors great enough so that 95 percent of the people who use the doors will have at least 1 foot clearance. Assuming that the heights are normally distributed with mean 70 inches and standard deviation 3 inches, how high must the doors be?

EXERCISE 6
A machine that dispenses ketchup into plastic bottles provides amounts of ketchup that follow the normal distribution with mean 28 ounces and standard deviation 0.8. The consumer protection agency requires that only 7% of all the bottles of ketchup can be under the weight that is written on the bottles.

a. What weight should be printed on the bottles of ketchup?

b. Suppose that this machine dispenses ketchup into bottles that are labeled as 25-ounce bottles. You can adjust the mean amount dispensed, but you do not know the standard deviation. Suppose that the machine is set to dispense 25.8 ounces of ketchup, and you find that 11% of the bottles are underweight. What is the standard deviation?

EXERCISE 7
One thousand observations are randomly selected from a population that follows the normal distribution. Using your knowledge on boxplots, binomial distribution and of course normal distribution how many of these observations are expected to be outliers (suspected or serious)?

EXERCISE 8
At a certain hospital there are 100 available beds for patients at any given day. The probability that a patient will stay at the hospital for further observation is 10%. Suppose that patients arrive independently of each other.

a. Write an expression of the exact probability that, if 1000 patients arrive at this hospital at any given day, there will not be enough space for all the patients that must stay for further observation.

b. Approximate the above probability using the normal distribution.

c. How many additional beds will the hospital need if they want the above probability to be only 2%?

EXERCISE 9

The chocolate chip cookies that are produced at a certain bakery have weights which are normally distributed with mean weight 18 grams and with standard deviation 3 grams.

a. The cookies, however, are sold by count, not by weight. This bakery wants to improve their image and they decide to set aside the lightest 20% of the cookies to be packaged and sold separately. What cookie weight will divide the lightest 20% from the heaviest 80%?

b. Suppose 4 cookies will be randomly selected. Give an interval that covers the middle 80% of the distribution of the sample mean.

c. What is the probability that a box with 36 such cookies will weigh more than 675 grams?

EXERCISE 10

Answer the following questions:

a. The height X of a certain type of trees at a park follows the normal distribution with mean 20 feet and standard deviation 2.5 feet. The annual cost Y (in dollars) for maintaining a tree of this type is a function of its height and it is equal to $Y = 5X + 2$. Find the probability that the cost for a randomly selected tree from this park will be between \$96 and \$115.

b. Ten trees (from question (a)) are randomly selected. Find the probability that two of them will have height more than 23 feet.

c. Find the 5_{th} percentile of the distribution of the total cost of 10 such trees from part (a).

EXERCISE 11

Suppose that X, Y, and Z are independent and each follows $N(0, 1)$. Find $P(3X + 2Y < 6Z - 7)$

EXERCISE 12

If we select two samples of size n_1 and n_2 from two normal populations with known variances σ_1^2 and σ_2^2 then the difference between the two sample means $(\bar{X}_1 - \bar{X}_2)$ follows the normal distribution as follows.

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

Suppose the distributions of X_1 and X_2 are $N(184, 40)$ and $N(172, 51)$ respectively. Find the 40th percentile of the distribution of $\bar{X}_1 - \bar{X}_2$ if 50 observations are randomly selected from each population $(n_1 = n_2 = 50)$.

EXERCISE 13

Let X and Y denote the wing lengths (in mm) of a male and female Hawaiian gallinule. Assume that the respective distributions of X and Y are $N(184, 40)$ and $N(172, 51)$. Note: $\sigma_X = 40, \sigma_Y = 51$.

a. If $\bar{X}$ and $\bar{Y}$ are the sample means of random samples of sizes $n_1 = n_2 = 16$ birds of each sex, find the probability that the sample mean of the wing length of the 16 male birds exceeds that one of the female birds by 9 mm. In other words, find $P(\bar{X} - \bar{Y} > 9)$.

b. Find the 5_{th} percentile of the distribution of $\bar{X} - \bar{Y}$.

c. What is the probability that among 5 female Hawaiian gallinules 2 will have wing length more than 200 mm?