University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Homework 4

EXERCISE 1
Suppose that 60% of Los Angeles residents support the idea of constructing a subway system that runs from Downtown Los Angeles to Santa Monica through Wiilshire Boulevard. What is the probability that in a random sample of 400 Los Angeles residents the sample proportion will be less than 53%?

EXERCISE 2
Information on a packet of seeds claims that the germination rate is 92%. What is the probability that more than 95% of the 160 seeds in a packet will germinate? Is there any problem with assumptions here?

EXERCISE 3
About 60% of restaurant customers demand a smoke-free environment and so they ask to be seated in a nonsmoking section. A new restaurant with 120 seats will starts its business very soon. How many seats should be in the nonsmoking area in order to be very sure that there is enough space for the nonsmokers. First comment on what "very sure" means.

EXERCISE 4
The carbon monoxide (CO) emissions for a certain kind of car follows a distribution with mean $\mu = 2.9gm/mi$ and standard deviation $\sigma = 0.4gm/mi$. Suppose that a company has in its fleet 80 of these cars.

a. What is the distribution of the sample mean ($\bar{X}$) of these 80 cars?

b. What is the probability that the sample mean is between 3.0 and 3.1 mg/mi?

c. Find the $5th$ percentile of the distribution of the sample mean.

EXERCISE 5
The amount of mineral water consumed by a person per day on the job is normally distributed with mean 19 ounces and standard deviation 5 ounces. A company supplies its employees with 2000 ounces of mineral water daily. The company has 100 employees.

a. Find the probability that the mineral water supplied by the company will not satisfy the water demanded by its employees.

b. Find the probability that in the next 4 days the company will not satisfy the water demanded by its employees on at least 1 of these 4 days. Assume that the amount of mineral water consumed by the employees of the company is independent from day to day.

c. Find the probability that during the next year (365 days) the company will not satisfy the water demanded by its employees on more than 15 days.

EXERCISE 6
You are going to play roulette at a casino. As a reminder a roulette has 38 numbers (1-36 plus 0 and 00). Let's say that you want to bet on number 13. If you win it pays $35:1$ which means that you get your \$1 back plus \$35.

a. In 10000 plays, what is the probability that the casino will make more than \$400?

b. What would the probability be if you play 90000 games?

c. Write the expression that computes the exact probability of (a). (Use binomial distribution).

EXERCISE 7
Suppose the mean cholesterol level for U.S. females between the ages of 20-30 is 180 `mg/dl` and the standard deviation is 32 `mg/dl`. A random sample of 64 women from this age group is selected. Find the probability that the sample mean cholesterol of these 64 women will be:

a. Larger than 185 `mg/dl`.

b. Less than 173 `mg/dl`.

c. Between 173 and 185 `mg/dl`.

EXERCISE 8
Consider the simple regression model through the origin: $y_i = \beta_1 x_i + \epsilon_i$. The error terms are independent and $\epsilon_i \sim N(0, \sigma)$. It was shown in homework 2, exercise 3 that $\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$. Find

a. $E(\hat{\beta}_1)$.

b. $var(\hat{\beta}_1)$.