University of California, Los Angeles
Department of Statistics

Statistics 13 — Instructor: Nicolas Christou

Homework 6

EXERCISE 1
Advertisements claim that the average nicotine content of a certain kind of cigarette is only 0.30 milligram. Assume that the population standard deviation is 0.15 milligram. Suspecting that this figure is too low, the health department of the City of Los Angeles and the consumer protection service take a random sample of 121 of those cigarettes from different production lots. They find that the sample mean is $\bar{x} = 0.33$ milligram.

a. Use the 0.05 level of significance to test if the nicotine content is more than 0.30 milligram.

b. With the same data using $\alpha = 0.10$ what is your conclusion. No calculations!

EXERCISE 2
The output voltage of a certain electric circuit is specified to be 130 volts. The population standard deviation is known to be $\sigma = 3.0$ volts. A sample of 40 readings on the voltage of this circuit gave a sample mean of 128.6 volts.

a. Test the hypothesis that the mean output voltage is 130 volts against the alternative that it is less than 130 volts. Use $\alpha = 0.05$.

b. Based on your answer to (a), is it possible that the mean output voltage is still 130 volts? Explain.

c. For this part you do not have to show any calculations.
How would the type II error β be affected if the type I error α decreases to 0.01?

EXERCISE 3
A manufacturer claims that 20% of the public preferred her product. A sample of 100 persons is taken to check her claim. It is found that 8 of these 100 persons preferred her product.

a. Using the 0.05 level of significance test her claim (perform a two-sided test).

b. Find the p-value and explain what it means.

EXERCISE 4
Investigating the possibility that a coin-operated soda vending machine is dispensing too much soda, the owner of the machine takes a random sample of 41 "6-fluid-ounce" servings and finds that the sample mean is $\overline{x} = 6.15$ fluid ounces with sample standard deviation $s = 0.3$ fluid ounces.

a. Is there evidence of overfilling? Use 0.01 level of significance.

b. Explain to a friend of yours who has not taken statistics yet, what would be a type I error and a type II error in testing the hypothesis of part (a).

EXERCISE 5
Last month, a large supermarket chain received many consumer complaints about the quantity of chips in 16-ounce bags of a particular brand of potato chips. Suspecting that the complaints were merely the result of the potato chips settling to the bottom of the bags during shipping, but wanting to be able to assure its customers they were getting their money's worth, the chain decided to test the following hypotheses concerning the mean weight (in ounces) of a bag of potato chips in the next shipment of potato chips received from their largest supplier:
$H_0 : \mu = 16$
$H_a : \mu < 16$
If there is evidence that $\mu < 16$, then the shipment would be refused and a complaint registered with the supplier.

a. What is a Type I error, in terms of the problem?

b. What is a Type II error, in terms of the problem?

c. Which type of error would the chain's customers view as more serious? Which type of error would the chain's supplier view as more serious?

EXERCISE 6
Find the $p-$value of the test in exercise 2 and explain what it means.

EXERCISE 7
Refer to exercise 1. If the actual population mean nicotine content is 0.31 milligram, compute the type II error β, and the power of the test $1 - \beta$.

EXERCISE 8
Refer to exercise 2. If the true population mean output voltage is 128.6 volts, compute the probability of a type II error (β) and the power of the test $(1 - \beta)$ when $\alpha = 0.05$.

EXERCISE 9
A coin is thrown independently 10 times to test that the probability of heads is $\frac{1}{2}$ against the alternative that the probability is not $\frac{1}{2}$. The test rejects H_0 if either 0 or 10 heads are observed.

a. What is the significance level α of the test?

b. If in fact the probability of heads is 0.1, what is the power of the test?

EXERCISE 10
Suppose that $X \sim bin(100, p)$. Consider the test that rejects $H_0 : p = 0.5$ in favor of $H_a : p \neq 0.5$ for $|X - 50| > 10$. Use the normal approximation to binomial distribution to answer the following:

a. What is α?

b. Graph the power as a function of p.

EXERCISE 11
True or false:

a. If the sample size is decreased when testing a hypothesis, the power would be expected to increase.

b. If the significance level of a test is decreased, the power would be expected to increase.

c. If a test is rejected at the significance level α, the probability that the null hypothesis is true equals α.

d. The probability that the null hypothesis is falsely rejected is equal to the power of the test.

e. A Type I error occurs when the test statistic falls in the rejection region of the test.

f. In testing a hypothesis, when the difference between the hypothesized mean and the actual mean (shift from μ_0 to μ_a) is increased, the power of the test will decrease.

g. In testing a hypothesis, when the standard deviation is decreased, the power of the test will decrease.

Example 12
An experimenter has prepared a drug dosage level that he claims will induce sleep for at least 80% of those people suffering from insomnia. After examining the dosage, we feel that his claims regarding the efectiveness of the dosage are inflated. In an attempt to disprove his claim, we administer his prescribed dosage to 20 insomniancs, and we observe X, the number having sleep induced by the drug dose. We wish to test the hypothesis $H_0 : p = 0.8$ against the alternative $H_a : p < 0.8$. Assume the rejection region $X \leq 12$ is used.

a. Find the type I error α.

b. Find the type II error β if the true $p = 0.6$.

c. Find the type II error β if the true $p = 0.4$.

Example 13
For a certain candidate's political poll $n = 15$ voters are sampled. Assume that this sample is taken from an infinite population of voters. We wish to test H_0: $p = 0.5$ against the alternative H_a: $p < 0.5$. The test statistic is X, which is the number of voters among the 15 sampled favoring this candidate.

a. Calculate the probability of a type I error α if we select the rejection region to be $RR = \{x \leq 2\}$.

b. Is our test good in protecting us from concluding that this candidate is a winner if, in fact, he will lose? Suppose that he really will win 30% of the vote ($p = 0.30$). What is the probability of a type II error β that the sample will erroneously lead us to conclude that H_0 is true?