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Statistics 13 **Instructor: Nicolas Christou**

Homework 6 - Solutions

EXERCISE 1

a. We follow the 4 steps:

1.

$$H_0: \quad \mu = 0.30$$
$$H_a: \quad \mu > 0.30$$

2. We compute the test statistic z:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{0.33 - 0.30}{\frac{0.15}{\sqrt{121}}} \Rightarrow z = 2.2$$

3. We find the rejection region. Here we use significance level $\alpha = 0.05$, therefore the rejection region will be when $z > 1.645$.

4. Conclusion: Since $z = 2.2 > 1.645$ we reject H_0. Therefore based on the evidence the average nicotine content is more than 0.30 milligram.

b. Reject H_0

EXERCISE 2

a. We follow the 4 steps:

1.

$$H_0: \quad \mu = 130$$
$$H_a: \quad \mu < 130$$

2. We compute the test statistic z:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{128.6 - 130}{\frac{3}{\sqrt{40}}} \Rightarrow z = -2.95$$

3. We find the rejection region. Here we use significance level $\alpha = 0.05$, therefore the rejection region will be when $z < -1.645$.

4. Conclusion: Since $z = -2.95 < -1.645$ we reject H_0. Therefore based on the evidence the mean output voltage is less than 130 volts.

b. Yes, it is possible that the mean output voltage is still 130 volts. We may have commited a type I error.

c. The Type II error β will increase.

EXERCISE 3

a. We follow the 4 steps:

1.

$$H_0: \quad p = 0.20$$
$$H_a: \quad p \neq 0.20$$

2. We compute the test statistic z:

$$z = \frac{p_s - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.08 - 0.20}{\sqrt{\frac{0.20(1-0.20)}{100}}} \Rightarrow z = -3.0$$

3. We find the rejection region. Here we use significance level $\alpha = 0.05$, and because this is a two-sided test we find that $z_{0.025} = 1.96$ and the rejection region is when $z > z_{0.025}$ or $z < -z_{0.025}$.

4. Conclusion: Since $z = -3.0 < -1.96$ we reject H_0. Therefore we reject the claim that 20% of the public prefer the product.

b. The p-value is the probability of observing the test statistic that we computed or a larger value. We computed $z = -3.0$. Therefore the p-value=$2P(z < -3.0) = 2(0.0013) \Rightarrow$ p-value= 0.0026. We multiply by 2 because this is a two-sided test.

EXERCISE 4
We are give $n = 41$, $\bar{x} = 6.15$ fluid ounces, and $s = 0.30$ fluid ounces.

a. We follow the 4 steps:

1.
$$H_0 : \quad \mu = 6$$
$$H_a : \quad \mu > 6$$

2. We compute the test statistic t:
$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{6.15 - 6}{\frac{0.30}{\sqrt{41}}} \Rightarrow t = 3.20$$

3. We find the rejection region. Here we use significance level $\alpha = 0.01$, and $n - 1 = 41 - 1 = 40$ degrees of freedom. From the t table we find that the rejection region is when the test statistic t is larger than $t_{0.01;40} = 2.423$.

4. Conclusion: Since $t = 3.20 > 2.423$ we reject H_0. Therefore this is an evidence of overfilling.

b. Type I error: Falsely conclude that there is evidence of overfilling. Type II error: Falsely conclude that there is not overfilling.

EXERCISE 5

a. A type I error in terms of this problem will be if we decide that the supplier gives less mean weight of potato chips when actually he gives the correct amount.

b. A type II error in terms of this problem will be if we decide that the supplier gives the correct mean amount of potato chips when actually he gives less.

c. The customers view as more serious a type II error beacuse no action will be taken by the supplier. The supplier views as more serious a type I error because unecessary action will be taken.

EXERCISE 6
The $p-$value is:
$p-$value: $p-$value$= P(z < -2.95) = 0.0016$. Since the $p-$value is less than α we reject the H_0 hypothesis. If $\mu = 130$ is true, then there is only 0.0016 probability of observing this sample mean or a more extreme value. This is a very unlikely event, therefore we reject H_0.

EXERCISE 7
First we want to find the values of $\bar{x}$ for which we reject H_0. From part (a) the rejection region is $z > 1.645$, Or

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} > 1.645 \Rightarrow \frac{\bar{x} - 0.30}{\frac{0.15}{\sqrt{121}}} > 1.645 \Rightarrow$$
$$\bar{x} - 0.30 > 1.645 \frac{0.15}{\sqrt{121}} \Rightarrow \bar{x} > 0.3224.$$

Therefore we reject H_0 if $\bar{x} > 0.3224$. Now, we want to compute the probability of commiting a type II error. This can be written as:
P(do not reject H_0 when in fact H_0 is not true) $=$
P($\bar{x} < 0.3224$ when the true mean is $\mu = 0.31$) $=$

$$P(z < \frac{0.3224 - 0.31}{\frac{0.15}{\sqrt{121}}}) = P(z < 0.91) = 0.5 + 0.3186$$

The type II error is $\beta = 0.8186$ and the power of the test is $1 - \beta = 1 - 0.8186 = 0.1814$. This is a low-power test. The probability of detecting the difference between the hypothesized and true mean is only 18.14%.

EXERCISE 8
Part(c). First we want to find the values of $\bar{x}$ for which we reject H_0. From part (a) the rejection region is $z < -1.645$, Or

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < -1.645 \Rightarrow \frac{\bar{x} - 130}{\frac{3}{\sqrt{40}}} < -1.645 \Rightarrow$$
$$\bar{x} - 130 < -1.645 \frac{3}{\sqrt{40}} \Rightarrow \bar{x} < 129.22$$

Therefore we reject H_0 if $\bar{x} < 129.22$. Now, we want to compute the probability of commiting a type II error. This can be written as:
P(do not reject H_0 when in fact H_0 is not true) $=$
P($\bar{x} > 129.22$ when the true mean is $\mu = 128.6$) $=$

$$P(z > \frac{129.22 - 128.6}{\frac{3}{\sqrt{40}}} = P(z > 1.31) = 0.5 - 0.4049 = 0.0951$$

The type II error is $\beta = 0.0951$ and the power of the test is $1 - \beta = 1 - 0.0951 = 0.9049$. This is a high-power test. The probability of detecting the difference between the hypothesized and true mean is 90.49%.

EXERCISE 9
Here we are testing:
$H_0 : p = \frac{1}{2}$
$H_a : p \neq \frac{1}{2}$

a. $\alpha = P(X = 0 \ or \ X = 10) = \binom{10}{0} \frac{1}{2}^0 \frac{1}{2}^{10} + \binom{10}{10} \frac{1}{2}^{10} \frac{1}{2}^0 = 0.0020$. Therefore the significance level is $\alpha = 0.0020$.

b. $1 - \beta = P(X = 0 \ or \ X = 10 | p = 0.10) = \binom{10}{0} 0.1^0 0.9^{10} + \binom{10}{10} 0.1^{10} 0.9^0 = 0.3487$. Therefore the power of the test is $1 - \beta = 0.3487$.

EXERCISE 10
We are given $X \sim bin(100, p)$ and we are testing
$H_0 : p = 0.50$
$H_a : p \neq 0.50$

a. We reject H_0 if $|X - 50| > 10$ which is the same as $X < 40$ *or* $X > 60$. For $n = 100$, $p = 0.50$ we have $\mu = np = 100(0.5) = 50$ and $\sigma = \sqrt{np(1-p)} = \sqrt{100(0.50)(1-0.50)} = 5$. The probability of falsely rejecting H_0 is $\alpha = P(X < 40) + P(X > 60) = P(Z < \frac{39.5-50}{5}) + P(Z > \frac{60.5-50}{5}) = P(z < -2.1) + P(z > 2.1) = 2(0.0179) = 0.0358$. Therefore $\alpha = 0.0358$. Note: We used the continuity correction for the normal approximation to binomial.

b. Find $1 - \beta = P(X < 40 \text{ or } X > 60|p)$ for different values of p, say $p = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. Then place the power on the vertical axis and p on the horizontal axis to graph the power as a function of p.

EXERCISE 11

a. False.

b. False.

c. False.

d. False.

e. False.

f. False.

g. False.

EXERCISE 12
We are testing:
$H_0 : p = 0.80$
$H_a : p < 0.80$

a. $$\alpha = P(\text{falsely rejecting } H_0) = P(X \leq 12) = \sum_{x=0}^{12} \binom{20}{x} 0.80^x 0.20^{20-x} = 0.0321.$$

b. $$\beta = P(\text{falsely accepting } H_0) = P(X > 12) = \sum_{x=13}^{20} \binom{20}{x} 0.60^x 0.40^{20-x} = 0.4159.$$

c. $$\beta = P(\text{falsely accepting } H_0) = P(X > 12) = \sum_{x=13}^{20} \binom{20}{x} 0.40^x 0.60^{20-x} = 0.0210.$$

EXERCISE 13
We are testing:
$H_0 : p = 0.50$
$H_a : p < 0.50$

a. $$\alpha = P(\text{falsely rejecting } H_0) = P(X \leq 2) = \sum_{x=0}^{2} \binom{15}{x} 0.50^x 0.50^{15-x} = 0.0037.$$

b. $$\beta = P(\text{falsely accepting } H_0) = P(X > 2) = \sum_{x=3}^{15} \binom{15}{x} 0.30^x 0.70^{15-x} = 0.8732.$$