

Binomial and Poisson distributions

• **Binomial probability distribution**

Suppose that  $n$  independent trials each one having probability of success  $p$  are to be performed. Let  $X$  be the number of successes among the  $n$  trials. We say that  $X$  follows the binomial probability distribution with parameters  $n, p$ .

Probability model:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, 2, 3, \dots, n$$

or

$$P(X = x) = nCx p^x (1 - p)^{n-x}, \quad x = 0, 1, 2, 3, \dots, n$$

where

$$nCx = \binom{n}{x} = \frac{n!}{(n-x)!x!}$$

$$\begin{aligned} \text{Expected value of } X: & \quad E(X) = np \\ \text{Variance of } X: & \quad \sigma^2 = np(1-p) \\ \text{Standard deviation of } X: & \quad \sigma = \sqrt{np(1-p)} \end{aligned}$$

• **Poisson probability distribution:**

The Poisson probability mass function with parameter  $\lambda > 0$  (where  $\lambda$  is the average number of events occur per time, area, volume, etc.) is:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

$$\begin{aligned} \text{Expected value of } X: & \quad E(X) = \lambda \\ \text{Variance of } X: & \quad \sigma^2 = \lambda \\ \text{Standard deviation of } X: & \quad \sigma = \sqrt{\lambda} \end{aligned}$$

Note:

1. Know how to use these functions to compute probabilities. Use them in some numerical examples for better understanding.
2. Under certain conditions, binomial and Poisson can be approximated by normal. When? Also use some numerical examples where you can approximate binomial and poisson with normal. In Monday's lab you will use R for binomial and normal distributions.