



- We can write:

$$\hat{p} \sim N \left( p, \sqrt{\frac{p(1-p)}{n}} \right)$$

Therefore

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- How can we use the above statement? Here is an example:  
Suppose the population proportion of Democrats among UCLA students is 55%. Find the probability that the sample proportion of 200 students randomly selected will exceed 60%.

- Important:  
The population proportion  $p$  is a parameter, it is a fixed number between 0 and 1. It is not random!  
The sample proportion  $\hat{p}$  is not fixed. It is random!



- We can write:

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Therefore

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- As a result of the previous statement the following is also true (and very useful):

$$T \sim N(n\mu, \sigma\sqrt{n})$$

Therefore

$$Z = \frac{T - n\mu}{\sigma\sqrt{n}}$$

- How can we use the above statement? Here is an example:

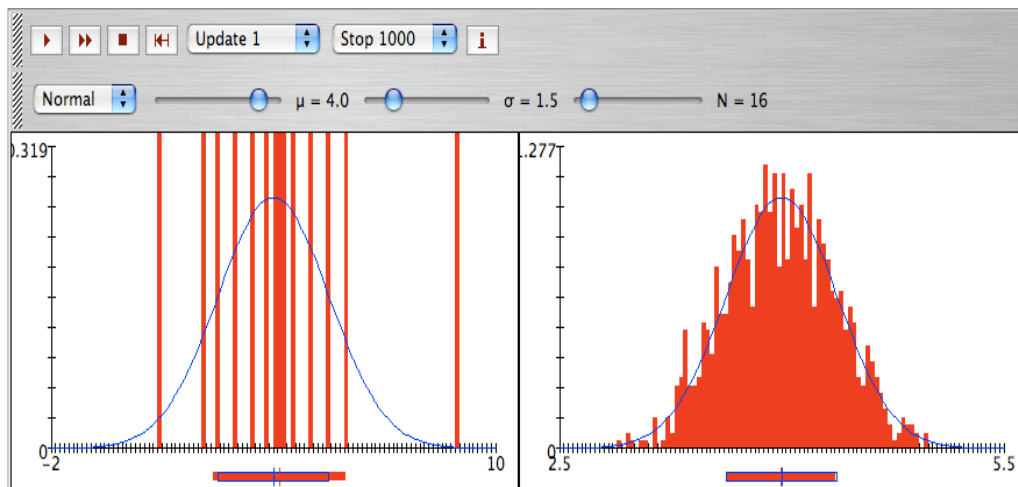
A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean  $\mu = 205$  pounds and standard deviation  $\sigma = 15$  pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?

- Important:

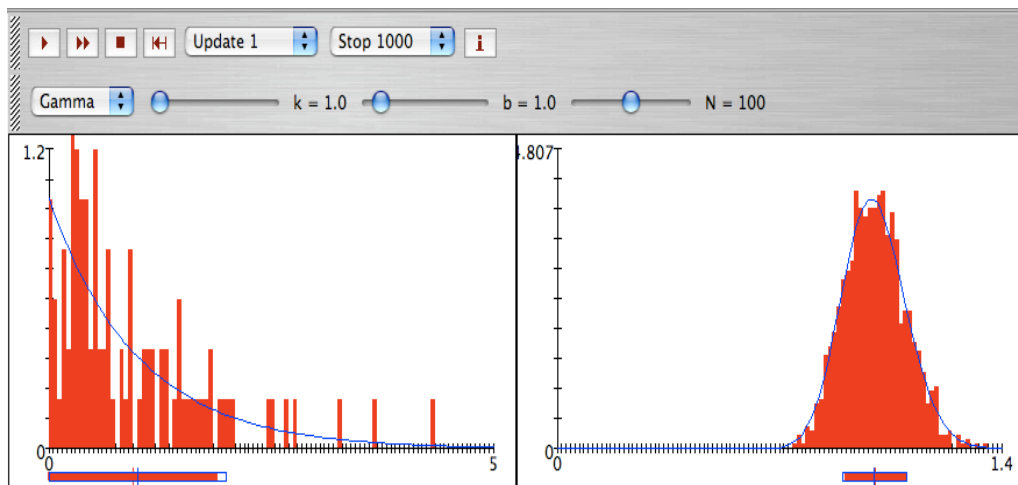
The population mean  $\mu$  is a parameter, it is a fixed number. It is not random!  
The sample mean  $\bar{X}$  is not fixed. It is random!

Here are two simulation experiments from the SOCR sample mean experiment:

A. Population  $N(4, 1.5)$ . Sample size  $n = 16$ . We observe that because the population from where the samples are selected is normal even a small sample (here  $n = 16$ ) will produce a bell-shaped distribution for the sample mean  $\bar{X}$ .

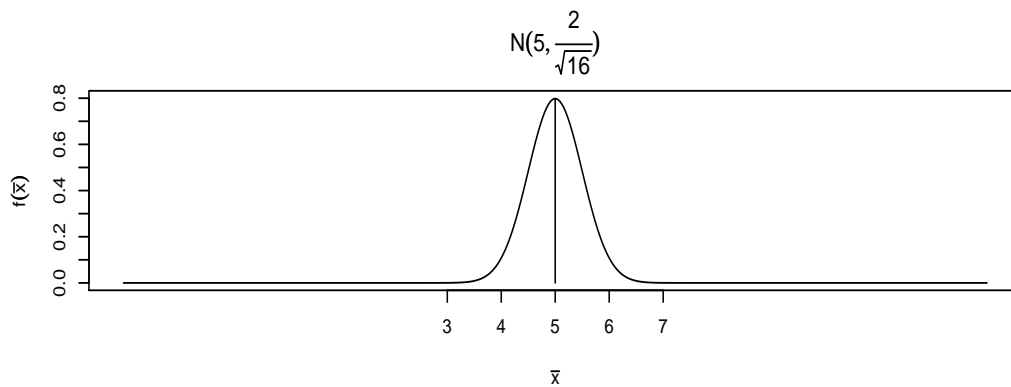
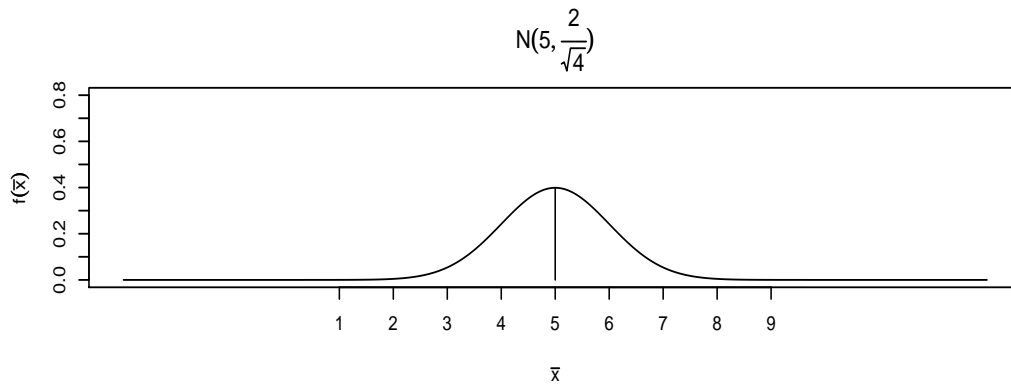
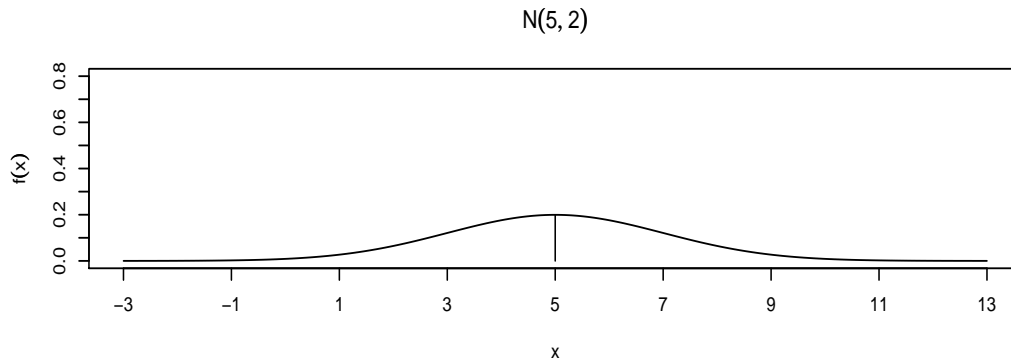


B. Population  $exp(\lambda = 1)$ . Sample size  $n = 100$ :



## Distribution of the sample mean - Sampling from normal distribution

If we sample from normal distribution  $N(\mu, \sigma)$  then  $\bar{X}$  follows exactly the normal distribution with mean  $\mu$  and standard deviation  $\frac{\sigma}{\sqrt{n}}$  regardless of the sample size  $n$ . In the next figure we see the effect of the sample size on the shape of the distribution of  $\bar{X}$ . The first figure is the  $N(5, 2)$  distribution. The second figure represents the distribution of  $\bar{X}$  when  $n = 4$ . The second figure represents the distribution of  $\bar{X}$  when  $n = 16$ .



## Sampling distributions - Examples

### Example 1

A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean  $\mu = 205$  pounds and standard deviation  $\sigma = 15$  pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?

### Example 2

From past experience, it is known that the number of tickets purchased by a student standing in line at the ticket window for the football match of *UCLA* against *USC* follows a distribution that has mean  $\mu = 2.4$  and standard deviation  $\sigma = 2.0$ . Suppose that few hours before the start of one of these matches there are 100 eager students standing in line to purchase tickets. If only 250 tickets remain, what is the probability that all 100 students will be able to purchase the tickets they desire?

### Example 3

Suppose that you have a sample of 100 values from a population with mean  $\mu = 500$  and with standard deviation  $\sigma = 80$ .

- What is the probability that the sample mean will be in the interval  $(490, 510)$ ?
- Give an interval that covers the middle 95% of the distribution of the sample mean.

### Example 4

The amount of mineral water consumed by a person per day on the job is normally distributed with mean 19 ounces and standard deviation 5 ounces. A company supplies its employees with 2000 ounces of mineral water daily. The company has 100 employees.

- Find the probability that the mineral water supplied by the company will not satisfy the water demanded by its employees.
- Find the probability that in the next 4 days the company will not satisfy the water demanded by its employees on at least 1 of these 4 days. Assume that the amount of mineral water consumed by the employees of the company is independent from day to day.
- Find the probability that during the next year (365 days) the company will not satisfy the water demanded by its employees on more than 15 days.

### Example 5

Supply responses *true* or *false* with an explanation to each of the following:

- The probability that the average of 20 values will be within 0.4 standard deviations of the population mean exceeds the probability that the average of 40 values will be within 0.4 standard deviations of the population mean.
- $P(\bar{X} > 4)$  is larger than  $P(X > 4)$  if  $X \sim N(8, \sigma)$ .
- If  $\bar{X}$  is the average of  $n$  values sampled from a normal distribution with mean  $\mu$  and if  $c$  is any positive number, then  $P(\mu - c \leq \bar{X} \leq \mu + c)$  decreases as  $n$  gets large.

### Example 6

An insurance company wants to audit health insurance claims in its very large database of transactions. In a quick attempt to assess the level of overstatement of this database, the insurance company selects at random 400 items from the database (each item represents a dollar amount). Suppose that the population mean overstatement of the entire database is \$8, with population standard deviation \$20.

- Find the probability that the sample mean of the 400 would be less than \$6.50.
- The population from where the sample of 400 was selected does not follow the normal distribution. Why?
- Why can we use the normal distribution in obtaining an answer to part (a)?
- For what value of  $\omega$  can we say that  $P(\mu - \omega < \bar{X} < \mu + \omega)$  is equal to 80%?
- Let  $T$  be the total overstatement for the 400 randomly selected items. Find the number  $b$  so that  $P(T > b) = 0.975$ .

### Example 7

A telephone company has determined that during nonholidays the number of phone calls that pass through the main branch office each hour follows the normal distribution with mean  $\mu = 80000$  and standard deviation  $\sigma = 35000$ . Suppose that a random sample of 60 nonholiday hours is selected and the sample mean  $\bar{x}$  of the incoming phone calls is computed.

- Describe the distribution of  $\bar{x}$ .
- Find the probability that the sample mean  $\bar{x}$  of the incoming phone calls for these 60 hours is larger than 91970.
- Is it more likely that the sample average  $\bar{x}$  will be greater than 75000 hours, or that one hour's incoming calls will be?

### Example 8

Assume that the daily *S&P* return follows the normal distribution with mean  $\mu = 0.00032$  and standard deviation  $\sigma = 0.00859$ .

- Find the  $75_{th}$  percentile of this distribution.
- What is the probability that in 2 of the following 5 days, the daily *S&P* return will be larger than 0.01?
- Consider the sample average *S&P* of a random sample of 20 days.
  - What is the distribution of the sample mean?
  - What is the probability that the sample mean will be larger than 0.005?
  - Is it more likely that the sample average *S&P* will be greater than 0.007, or that one day's *S&P* return will be?

**Example 9**

Assume that 30% of students at a university wear contact lenses.

- a. We randomly pick 100 students. Describe the distribution of  $\hat{p}$  (the sample proportion of the 100 students who wear contact lenses). Do the requirements hold?
- b. What is the probability that more than  $\frac{1}{3}$  of this sample wear contact lenses?

**Example 10**

A restaurant owner anticipates serving about 180 people on Friday evening, and believing that about 20% of the customers will order the "special" of the day. How many of those special meals should he plan on serving in order to be very sure of having enough specials on hand to meet customer demand? First, what "very sure" means to you?

**Example 11**

You are going to play roulette at a casino. As a reminder a roulette has 38 numbers (1-36 plus 0 and 00). Let's say that you want to bet on number 13. If you win it pays 35 : 1 which means that you get your \$1 back plus \$35.

- a. In 10000 plays, what is the probability that the casino will make more than \$400?
- b. What would the probability be if you play 90000 games?

**Example 12**

Suppose you select a sample of size  $n = 16$  from a population that follows the normal distribution with  $\mu = 5$  and  $\sigma = 2$ .

- a. Describe the distribution of the sample mean  $\bar{X}$ .
- b. Find the probability that the sample mean will exceed 6.5.
- c. Describe the distribution of the total  $T$ .
- d. What is the probability that the total will fall between 80 to 100.

**Example 13**

Write a summary of the sampling distribution of  $\hat{p}$ ,  $\bar{X}$ , and  $T$ . What distribution do they follow? Draw these distributions. What are the requirements?

# Sampling distributions - Examples

## Answers

**EXERCISE 1**  
0.0099

**EXERCISE 2**  
0.6915

**EXERCISE 3**  
a. 0.7888  
b. (484.32, 515.68)

**EXERCISE 4**  
a. 0.0228  
b. 0.0881  
c. 0.0059

**EXERCISE 5**  
a. False  
b. True  
c. False

**EXERCISE 6**  
a. 0.0668  
b.  $X > 0, \sigma > \mu$   
c. large  $n$   
d. 1.28  
e. 2416

**EXERCISE 7**  
a.  $\bar{X} \sim N(80000, \frac{35000}{\sqrt{60}})$   
b. 0.0040  
c.  $P(\bar{X} > 75000)$  is larger..

**EXERCISE 8**  
a. 0.0061  
b. 0.1102  
c. i.  $\bar{x} \sim N(0.00032, \frac{0.00859}{\sqrt{20}})$   
ii. 0.0073  
iii. One day's  $S\&P$  will be larger

**EXERCISE 9**  
a.  $\hat{p} \sim N(0.30, \sqrt{\frac{0.30(1-0.30)}{100}})$   
b. 0.2327

**EXERCISE 10**  
 $\approx 45$

**EXERCISE 11**  
a. 0.5871  
b. 0.9940

**EXERCISE 12**  
a.  $\bar{X} \sim N(5, \frac{2}{\sqrt{16}})$   
b. 0.0013  
c.  $T \sim N(16(5), 2\sqrt{16})$   
d. 0.4938

**EXERCISE 13**  
Please see your classnotes!

## Review problems

### Example 1:

The length of time required for the periodic maintenance of an automobile is a random variable that follows the normal distribution with mean  $\mu = 1.4$  hours and standard deviation  $\sigma = 0.7$  hour.

- Find the 95<sup>th</sup> percentile of the above distribution.
- Suppose that the service department plans to service 50 automobiles per day. The department has 10 employees and each one works for 8 hours per day. What is the probability that the service department will have to work overtime?

### Example 2:

An airline, believing that 5% of passengers fail to show up for flights, overbooks (sells more tickets than there are seats). Suppose a plane will hold 265 passengers, and the airline sells 275 seats. Let  $X$  be the number of passengers that show up.

- What is the distribution of  $X$ ?
- Write an expression for the *exact* probability that the airline will not have enough seats for the passengers that show up.
- Approximate* the above probability using the normal distribution.

### Example 3:

We have discussed in class that if we select two samples of size  $n_1$  and  $n_2$  from two normal populations with known variances  $\sigma_1^2$  and  $\sigma_2^2$  then the difference between the two sample means ( $\bar{X}_1 - \bar{X}_2$ ) follows the normal distribution as follows.

$$\bar{X}_1 - \bar{X}_2 \sim N \left( \mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$

The corresponding  $z$  score is:

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- Let  $X$  and  $Y$  denote the wing lengths (in  $mm$ ) of a male and female Hawaiian gallinule. Assume that the respective distributions of  $X$  and  $Y$  are  $N(184, 40)$  and  $N(172, 51)$ . Note:  $\sigma_X = 40$ ,  $\sigma_Y = 51$ . If  $\bar{X}$  and  $\bar{Y}$  are the sample means of random samples of sizes  $n_1 = n_2 = 16$  birds of each sex, find the probability that the sample mean of the wing length of the 16 male birds exceeds that one of the female birds by 9  $mm$ . In other words, find  $P(\bar{X} - \bar{Y} > 9)$ .
- Find the 5<sup>th</sup> percentile of the distribution of  $\bar{X} - \bar{Y}$ .
- What is the probability that among 5 female Hawaiian gallinules 2 will have wing length more than 200  $mm$ ?