# University of California, Los Angeles Department of Statistics

## Statistics 13

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## Central limit theorem - Some problems

### Problem 1

## Part A:

It is claimed that the histogram below shows the distribution of the sample mean  $\bar{x}$ , when repeated samples of size n = 36 are selected with replacement from the population (2.6, 2.8, 3.0, 3.2, 3.4). Clearly explain if there is anything wrong with this histogram.

#### Part B:

- a. What distribution does the sum (total) of 36 observations selected from the same population as above follow?
- b. Sketch the histogram (roughly) of the total of repeated samples (with repcalcement) of size 36 selected from the above population. Make sure that you mark off some important values on the horizontal axis.
- c. FInd the  $5_{th}$  percentile of the distribution of T.



# Problem 2

A random sample of size n = 100 is selected form a distribution with mean  $\mu = 16$  and standard deviation  $\sigma = 4$ . Which one of the graphs below represents the distribution of the sample mean. Please explain your answer.



16.0 <del>x</del> 17.0

16.5

15.5

15.0

# Problem 3

Below you see the probability density function of an exponential distribution with parameter  $\lambda = \frac{1}{2}$ . It is claimed that the second graph is the distribution of the total T when a sample of size n = 100 is selected. Is there anything wrong with this graph?



### The distribution of the sample mean and the Central Limit Theorem An empirical investigation

The central limit theorem states that if a large sample of size n is selected from a population that has mmean  $\mu$  and standard deviation  $\sigma$  then the sample mean  $\bar{X}$  follows approximately:

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

In this experiment we will roll a die n = 80 times. Using the 80 outcomes we will compute the sample mean. We will repeat until we obtain 500 values of  $\bar{x}$ . At the end we will construct the histogram using the values of  $\bar{x}$ . Here is the population:

 $\begin{array}{cccc} X & P(X) \\ \hline 1 & \frac{1}{6} \\ 2 & \frac{1}{6} \\ 3 & \frac{1}{6} \\ 4 & \frac{1}{6} \\ 5 & \frac{1}{6} \\ 6 & \frac{1}{6} \end{array}$ 

This population has  $\mu = 3.5$  and  $\sigma = 1.71$ . Why?

The summary statistics of the 500 sample means:

. summarize xbar

Variable	Obs	Mean	Std. Dev.	Min	Max
xbar	500	3.499725	. 1822218	3.025	4

And below you can see the histogram of these 500 sample means:

