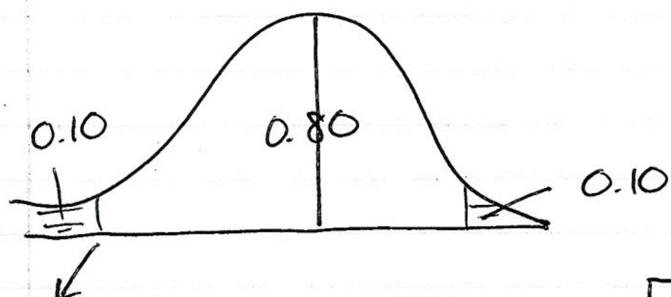
Exam 2 Practice
10/28/09

1a) $P(\mu - 5 \le \bar{x} \le \mu + 5) \ge 0.80$

$\sigma = 40 \quad n = ?$

$$P\left[\frac{\mu - 5 - \mu}{40/\sqrt{n}} \le Z \le \frac{\mu + 5 - \mu}{40/\sqrt{n}}\right] \ge 0.80$$

$$P\left(-\frac{\sqrt{n}}{8} \le Z \le \frac{\sqrt{n}}{8}\right) \ge 0.80$$



$$Z = -1.285 = -\frac{\sqrt{n}}{8} \Rightarrow \boxed{n = 106}$$

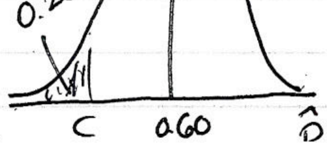
b. $P(\bar{x} > 7.5) = P\left(Z > \frac{7.5 - 7}{2.42/\sqrt{50}}\right)$

$P(Z > 1.46) = 1 - 0.9279 = 0.0721$

binomial $P(Y = 3) = \binom{5}{3}(0.0721^3)(1 - 0.0721)^2 = 0.0032$

c) $p = 0.60 \qquad P(\hat{p} > c) = 0.80$

c is the 20th percentile!



$$-0.845 = \frac{c - 0.6}{\sqrt{0.60(1-0.60}} \qquad \boxed{c = 0.58}$$

2) $N(16, 5.2)$

(a) $n_1 = 16$ $\quad$ $n_2 = \cancel{4}9$ $\quad$ $n_3 = 4$

$$\bar{X}_1 \sim N\left(16, \frac{5.2}{\sqrt{16}}\right) \qquad \bar{X}_2 \sim N\left(16, \frac{5.2}{\sqrt{9}}\right) \qquad \bar{X} \sim N\left(16, \frac{5.2}{\sqrt{4}}\right)$$

(b) $2\bar{X}_1 - \bar{X}_2 - \frac{1}{2}\bar{X}_3$

$$E\left(2\bar{X}_1 - \bar{X}_2 - \frac{1}{2}\bar{X}_3\right)$$

$$= 2E\bar{X}_1 - E\bar{X}_2 - \frac{1}{2}E\bar{X}_3$$

$$= 2(16) - 16 - \frac{1}{2}(16) = \boxed{8}$$

$$\mathrm{var}\left(2\bar{X}_1 - \bar{X}_2 - \frac{1}{2}\bar{X}_3\right)$$

$$= 4\,\mathrm{var}(\bar{X}_1) + \mathrm{var}(\bar{X}_2) + \frac{1}{4}\mathrm{var}(\bar{X}_3)$$

$$= \underbrace{4 \cdot \frac{5.2^2}{16} + \frac{5.2^2}{9} + \frac{1}{4} \cdot \frac{5.2^2}{4}}_{X} \quad \text{(sum up)}$$

reminder:

$$\mathrm{var}(\bar{x}) = \frac{\sigma^2}{n}$$

$$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

answer: $2\bar{X}_1 - \bar{X}_2 - \frac{1}{2}\bar{X}_3 \sim N\left(8, \sqrt{X}\right)$

Problem 3

(a) $n = 20{,}000$

$p = 0.00464$

$$P(x > 100) = \sum_{x=101}^{20000} \binom{20{,}000}{x} p^x (1-p)^{20000-x}$$

(b) $\mu = np = 92.8$

$\sigma = \sqrt{np(1-p)} = 9.6$

approximate using normal distribution:

$$P(x > 100) = P\left(z > \frac{100.5 - 92.8}{9.6}\right)$$

$$= P(z > 0.8) = 1 - 0.7881 = \boxed{0.2119}$$

(c) use graphs

dem 4b) $X \sim N(100, 3)$

$E(A) = E(X \cdot 3X)$

$= E(3X^2) = 3E(X^2)$

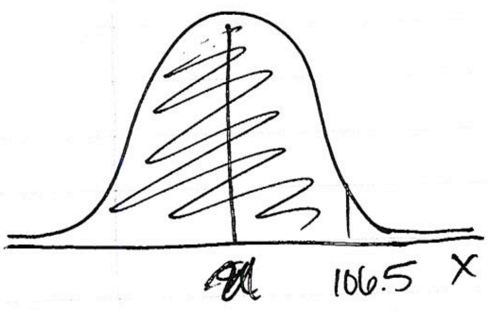
$= 3(\sigma^2 + \mu^2)$

$= 3(9 + 10{,}000) = \boxed{30{,}027}$

4c) $\mu = 10$ months
$\sigma = 10$ month

answer: <u>not</u> normal b/c n is small
parent population not stated as normal.

4d) $\mu = ?$ $\sigma = 3$ meters



$z = 1.885 = \dfrac{106.5 - \mu}{3}$

$\boxed{\mu = 100.845}$