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Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Hypothesis testing for proportion

Alternative Hypothesis H_a	Reject H_0 if
$p < p_0$	$Z < -Z_\alpha$
$p > p_0$	$Z > \ Z_\alpha$
$p \neq p_0$	$Z < -Z_{\alpha/2}$ or $Z > \ Z_{\alpha/2}$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Example 1
An experimenter has prepared a drug dosage level that he claims will induce sleep for at least 80% of those people suffering from insomnia. After examining the dosage, we feel that his claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove his claim, we administer his prescribed dosage to 20 insomniacs, and we observe X, the number having sleep induced by the drug dose. We wish to test the hypothesis $H_0 : p = 0.8$ against the alternative $H_a : p < 0.8$. Assume the rejection region $X \leq 12$ is used.

a. Find the type I error α.

b. Find the type II error β if the true $p = 0.6$.

c. Find the type II error β if the true $p = 0.4$.

Example 2
For a certain candidate's political poll $n = 15$ voters are sampled. We wish to test H_0: $p = 0.5$ against the alternative H_a: $p < 0.5$. The test statistic is X, which is the number of voters among the 15 sampled favoring this candidate.

a. Calculate the probability of a type I error α if we select the rejection region to be $RR = \{x \leq 2\}$.

b. Is our test good in protecting us from concluding that this candidate is a winner if, in fact, he will lose? Suppose that he really will win 30% of the vote ($p = 0.30$). What is the probability of a type II error β that the sample will erroneously lead us to conclude that H_0 is true?