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Statistics 13 **Instructor: Nicolas Christou**

Linear combinations of normally distributed random variables

Theory:

A. Let $X \sim N(\mu, \sigma)$. Then the random variable $Y = \alpha X + \beta$ is also normally distributed as follows:

$$Y \sim N(\alpha\mu + \beta, \alpha\sigma)$$

B. Let $X \sim N(\mu_X, \sigma_X)$ and $Y \sim N(\mu_Y, \sigma_Y)$. Then, if X and Y are independent, the random variable $S = X + Y$ follows also the normal distribution with mean $\mu_X + \mu_Y$ and standard deviation $\sqrt{\sigma_X^2 + \sigma_Y^2}$. We write

$$X + Y \sim N\left(\mu_X + \mu_Y, \sqrt{\sigma_X^2 + \sigma_Y^2}\right).$$

Also

$$aX + bY \sim N\left(a\mu_X + b\mu_Y, \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}\right)$$

and

$$aX - bY \sim N\left(a\mu_X - b\mu_Y, \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}\right)$$

and

$$aX + bY + c \sim N\left(a\mu_X + b\mu_Y + c, \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}\right)$$

Similarly, the random variable $D = X - Y$ follows

$$X - Y \sim N\left(\mu_X - \mu_Y, \sqrt{\sigma_X^2 + \sigma_Y^2}\right)$$

These results can be extended to more than 2 random variables. For example, if $X \sim N(\mu_X, \sigma_X)$, $Y \sim N(\mu_Y, \sigma_Y)$, and $Z \sim N(\mu_Z, \sigma_Z)$ then

$$X + Y + Z \sim N\left(\mu_X + \mu_Y + \mu_Z, \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}\right)$$

and also

$$aX + bY + cZ \sim N\left(a\mu_X + b\mu_Y + c\mu_Z, \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2 + c^2\sigma_Z^2}\right)$$

Example 1:
If the temperature in degrees Fahrenheit at a certain location is normally distributed with mean 68 degrees and a standard deviation of 4 degrees, what it the distribution of the temperature in degrees Celsius at the same location? It is given that $C = (F - 32)\frac{5}{9}$.

Example 2:
Suppose that the heights, in inches, of the women in a certain population follow the normal distribution with mean 65 inches and standard deviation 1 inch. The heights of men in this population follow the normal distribution with mean 68 inches and standard deviation 2 inches. One woman is selected at random and, independently, one man is selected at random. Find the probability that the woman will be taller than the man.

Example 3:
Suppose $X_1 \sim N(16, 5.2)$, $X_2 \sim N(16, 5.2)$, $X_3 \sim N(16, 5.2)$

a. What is the distribution of $X_1 + X_2 + X_3$?

b. What is the distribution of $\frac{X_1+X_2+X_3}{3}$?