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Statistics 13 **Instructor: Nicolas Christou**

Poker Probabilities

A Poker hand consists of five cards. Players try for combinations of two or more cards of a kind, five-card sequences, or five cards of the same suit. Poker is played with a standard 52-card deck in which all suits are of equal value, the cards ranking from the ace high, downward through king, queen, jack, and the numbered cards 10 to the deuce. The ace may also be considered low to form a straight (sequence) ace through five as well as high with king-queen-jack-10.

Rank of hands. The traditional ranking is:

1. Royal Flush (ace-king-queen-jack-ten all of the same suit).
2. Straight Flush (five cards of the same suit in sequence other than Royal Flush).
3. Four of a kind, plus any fifth card.
4. Full house (three of a kind plus a pair).
5. Flush (any five cards of the same suit other than straight flush).
6. Straight (five cards in sequence not all of the same suit).
7. Three of a kind plus no pair.
8. Two pairs plus any fifth card.
9. One pair plus three other cards.
10. Nothing

Rank	Poker Hand	# of Combinations	Probability
1.	Royal Flush	4	0.00000154
2.	Other Straight Flush	36	0.00001385
3.	Four of a kind	624	0.00024010
4.	Full House	3744	0.00144058
5.	Flush	5108	0.00196540
6.	Straight	10200	0.00392465
7.	Three of a kind	54912	0.02112845
8.	Two Pairs	123552	0.04753902
9.	One Pair	1098240	0.42256903
10.	Nothing	1302540	0.50117739
	Total	2598960	1.00000000

A standard 52-card deck consists of the following cards (13 denominations - 4 cards each, 4 suits - 13 cards each):

$\clubsuit$	$\spadesuit$	$\diamondsuit$	$\heartsuit$
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
J	J	J	J
Q	Q	Q	Q
K	K	K	K
A	A	A	A

There are $\binom{52}{5} = \frac{52!}{(52-5)!5!} = 2598960$ different ways to choose 5 cards from the available 52 cards. Let's compute the probability of the following poker hand: four of kind plus any fifth card: We need 2 different denominations (for example 4 aces plus an eight). There are $\binom{13}{2}$ different ways to choose 2 denominations from the 13 possible. Now, there are $\binom{4}{4}$ ways to choose the 4 aces from the 4 aces, and $\binom{4}{1}$ different ways to choose one eight from the 4 eights. Also, the opposite could have occured (that is 4 eights and 1 ace). Therefore the probability of a four of a kind plus any fifth card is:

$$P(\text{four of a kind}) = \frac{\binom{13}{2}\binom{4}{4}\binom{4}{1}2}{\binom{52}{5}} = 0.00024010.$$

The probability of a pair plus 3 other cards (for example 2 aces plus 1 two, 1 seven, 1 eight). We need 4 denominations. There are $\binom{13}{4}$ different ways to choose 4 denominations from the possible 13. Now, there are $\binom{4}{2}$ different ways to choose the pair from one of the 4 denominations, and $\binom{4}{1}$ different ways to choose one card from each of the other 3 denominations. Also, we multiply by 4 because the pair could have come from the 2s or the 7s or the 8s. Therefore the probability of a pair is:

$$P(\text{pair}) = \frac{\binom{13}{4}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}4}{\binom{52}{5}} = 0.42256903.$$