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Department of Statistics

Statistics 13

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Portfolio risk and return

An investor has a certain amount of dollars to invest into two stocks (*IBM* and *TEXACO*). A portion of the available funds will be invested into IBM (denote this portion of the funds with a) and the remaining funds into TEXACO (denote it with b) - so $a + b = 1$. The resulting portfolio will be $aX + bY$, where X is the monthly return of *IBM* and Y is the monthly return of *TEXACO*. The goal here is to find the most efficient portfolios given a certain amount of risk. Using historical market data we compute that $E(X) = 0.010$, $E(Y) = 0.013$, $Var(X) = 0.0061$, $Var(Y) = 0.0046$, and $Cov(X, Y) = 0.00062$.

We first want to minimize the variance of the portfolio. This will be:

$$\begin{aligned} &\text{Minimize } Var(aX + bY) \\ &\text{subject to } a + b = 1 \end{aligned}$$

Or

$$\begin{aligned} &\text{Minimize } a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y) \\ &\text{subject to } a + b = 1 \end{aligned}$$

Therefore our goal is to find a and b , the percentage of the available funds that will be invested in each stock. Substituting $b = 1 - a$ into the equation of the variance we get

$$a^2 Var(X) + (1 - a)^2 Var(Y) + 2a(1 - a)Cov(X, Y)$$

To minimize the above expression we take the derivative with respect to a , set it equal to zero and solve for a . The result is:

$$a = \frac{Var(Y) - Cov(X, Y)}{Var(X) + Var(Y) - 2Cov(X, Y)}$$

and therefore

$$b = \frac{Var(X) - Cov(X, Y)}{Var(X) + Var(Y) - 2Cov(X, Y)}$$

The values of a and b are:

$$a = \frac{0.0046 - 0.0062}{0.0061 + 0.0046 - 2(0.00062)} \Rightarrow a = 0.42.$$

and $b = 1 - a = 1 - 0.42 \Rightarrow b = 0.58$. Therefore if the investor invests 42% of the available funds into *IBM* and the remaining 58% into *TEXACO* the variance of the portfolio will be minimum and equal to:

$$Var(0.42X + 0.58Y) = 0.42^2(0.0061) + 0.58^2(0.0046) + 2(0.42)(0.58)(0.00062) = 0.002926.$$

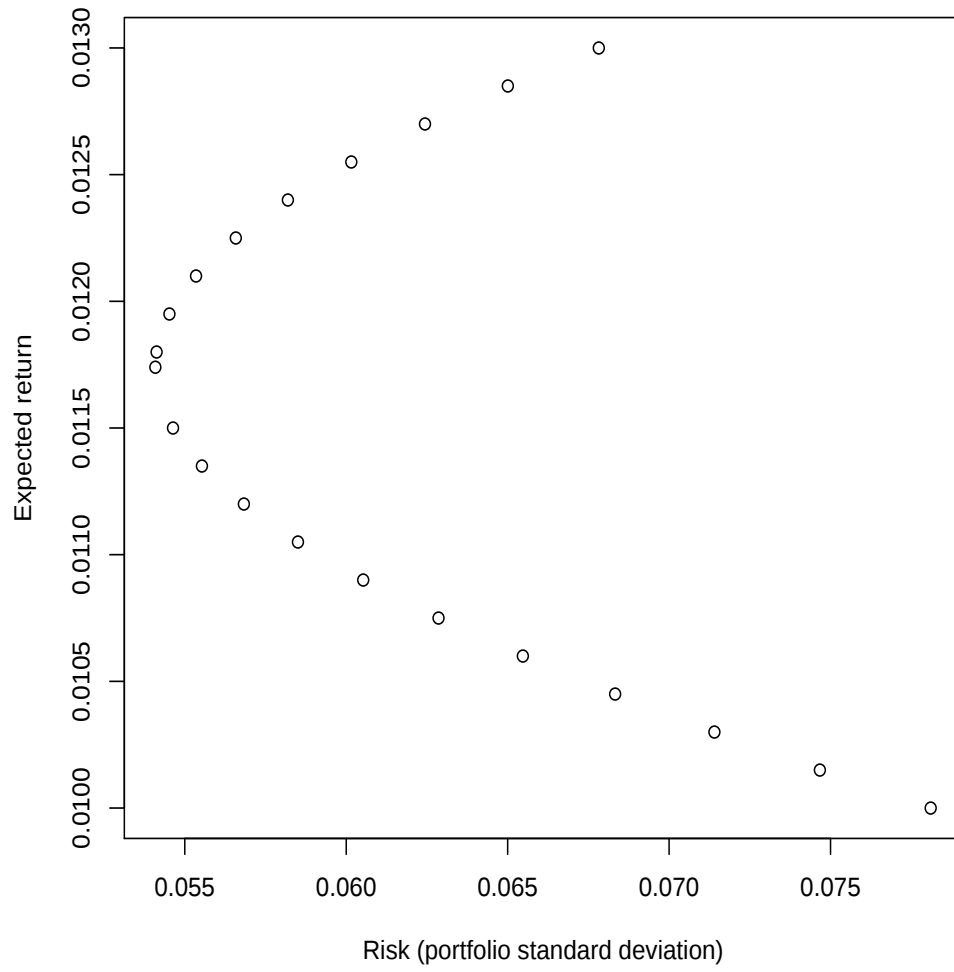
The corresponding expected return of this portfolio will be:

$$E(0.42X + 0.58Y) = 0.42(0.010) + 0.58(0.013) = 0.01174.$$

We can try many other combinations of a and b (but always $a + b = 1$) and compute the risk and return for each resulting portfolio. This is shown in the table below and the graph of return against risk on the other side.

a	b	Risk	Return
1.00	0.00	0.006100	0.01000
0.95	0.05	0.005576	0.01015
0.90	0.10	0.005099	0.01030
0.85	0.15	0.004669	0.01045
0.80	0.20	0.004286	0.01060
0.75	0.25	0.003951	0.01075
0.70	0.30	0.003663	0.01090
0.65	0.35	0.003423	0.01105
0.60	0.40	0.003230	0.01120
0.55	0.45	0.003084	0.01135
0.50	0.50	0.002985	0.01150
0.42	0.58	0.002926	0.01174
0.40	0.60	0.002930	0.01180
0.35	0.65	0.002973	0.01195
0.30	0.70	0.003063	0.01210
0.25	0.75	0.003201	0.01225
0.20	0.80	0.003386	0.01240
0.15	0.85	0.003619	0.01255
0.10	0.90	0.003899	0.01270
0.05	0.95	0.004226	0.01285
0.00	1.00	0.004600	0.01300

Portfolio possibilities curve

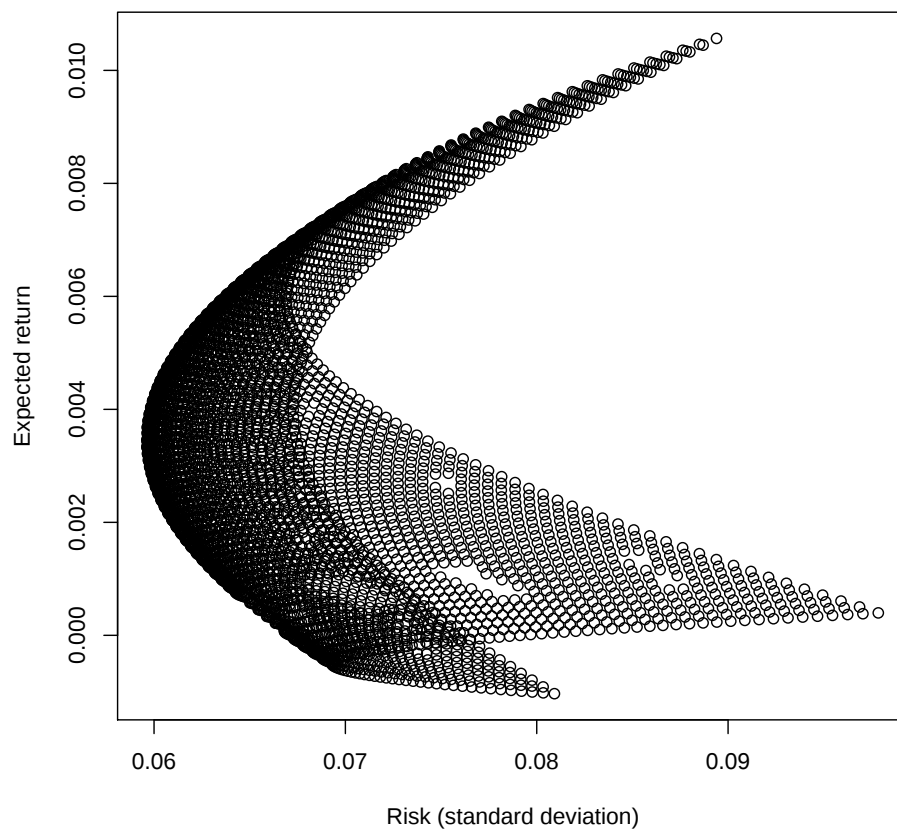


Efficient frontier with three stocks

```
> summary(returns)
      ribm      rxom      rboeing
Min.   :-0.2264526 Min.   :-0.5219233 Min.   :-0.34570
1st Qu.: -0.0515524 1st Qu.: -0.0172273 1st Qu.: -0.04308
Median : -0.0089916 Median :  0.0007013 Median :  0.01843
Mean   :  0.0003073 Mean    :-0.0011666 Mean    :  0.01079
3rd Qu.:  0.0462550 3rd Qu.:  0.0337488 3rd Qu.:  0.07357
Max.    :  0.3537987 Max.    :  0.2269380 Max.    :  0.17483

> cov(returns)
      ribm      rxom      rboeing
ribm   9.930174e-03 0.001798962 3.020685e-05
rxom   1.798962e-03 0.006743820 1.781462e-03
rboeing 3.020685e-05 0.001781462 8.282167e-03
```

Portfolio possibilities curve with 3 stocks



```

#==> Example using stockPortfolio <==#
# load package
library(stockPortfolio)

# select stocks
#IBM: International Business Machines Copr., WFC: Wells Fargo & Compnay
#JPM: JPMorgan Chase & Company, LUV: Southwest Airlines Co.
#XOM: Exxon Mobil Corporation, C: Citigroup, Inc.

ticker <- c("IBM", "WFC", "JPM", "LUV", "XOM", "C")

# get stock data
gr <- getReturns(ticker, start="2005-03-31", end="2010-03-31")

# gr is a "list" object and we find what it contains
# by typing the following:
names(gr)

# We can access each component of gr by typing:
gr$R

# obtain the variance-covariance matrix of the returns:
cov(gr$R)

# obtain the correlation matrix of the returns:
cor(gr$R)

# We can find summary statistics as follows:
summary(gr$R)

# To find the mean, variance, and standard deviation of a particular stock
mean(gr$R[,4])
var(gr$R[,4])
sd(gr$R[,4])

# To find the covariance and correlation between two stocks:
cov(gr$R[,4], gr$R[,6])
cor(gr$R[,4], gr$R[,6])

# Let's work with two stocks: IBM and LUV. Find the composition of the minimum risk portfolio:
x_IBM <- (var(gr$R[,4]) - cov(gr$R[,1], gr$R[,4])) /
(var(gr$R[,1]) + var(gr$R[,4]) - 2*cov(gr$R[,1], gr$R[,4]))
x_LUV <- 1-x_IBM

# Find the mean and sd of the minimum risk portfolio:
mean_min <- x_IBM*mean(gr$R[,1]) + x_LUV*mean(gr$R[,4])

var_min <- x_IBM^2*var(gr$R[,1]) + x_LUV^2*var(gr$R[,4]) +
2*x_IBM*x_LUV*cov(gr$R[,1], gr$R[,4])
sd_min <- var_min^0.5

# Construct the portfolio possibilities curve and identify the efficient frontier:
a <- seq(0,1,0.01)
b <- 1-a

mean_p <- a*mean(gr$R[,1]) + b*mean(gr$R[,4])

var_p <- a^2*var(gr$R[,1]) + b^2*var(gr$R[,4]) +
2*a*b*cov(gr$R[,1], gr$R[,4])
sd_p <- var_p^0.5

plot(sd_p,mean_p, type="l", xlab="Portfolio standard deviation (risk)", ylab="Portfolio expected return")
points(sd_min, mean_min, pch=19, col="green")

# Identify the efficient frontier:
xx <- cbind(sd_p,mean_p)
xxx <- xx[which(xx[,2]>mean_min),]
points(xxx, type="l", col="blue", lwd=3)

```