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Department of Statistics

Statistics 13

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Practice questions

**EXERCISE 1**

To determine whether or not they have a certain disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people. If the test is positive each of the 10 people will also be individually tested. Suppose the probability that a person has the disease is 0.10 for all people independently from each other.

- a. What is the probability that for the group of 10 people only one test is needed?
- b. What is the probability that for the group of 10 people 11 tests are needed? Why 11?
- c. Compute the expected number of tests necessary for each group of 10 people..

**EXERCISE 2**

In exercise 1, the 100 people were placed in groups of 10. Try different group sizes in order to minimize the expected number of tests? For example, you can try 2, 4, 5, 20, 25, 50. What group size gives the least expected number of tests?

**EXERCISE 1**

Let  $X$  be the number of tests needed for each group of 10 people. Then, if nobody has the disease 1 test is enough. But if the test is positive then there will be 11 test ( $1 + 10$ ). The probability distribution of  $X$  is:

$X$	$P(X)$
1	$0.90^{10}$
11	$1 - 0.90^{10}$

Therefore the expected number of tests is:

$$E(X) = 1(0.90)^{10} + 11(1 - 0.90^{10}) = 7.51.$$

**EXERCISE 2**

When  $n = 2$ ,  $E(X) = 1.38$ . The total number of tests for the 100 people is  $1.38(50) = 69$ .  
When  $n = 4$ ,  $E(X) = 2.38$ . The total number of tests for the 100 people is  $2.38(25) = 59.5$ .  
When  $n = 5$ ,  $E(X) = 3.05$ . The total number of tests for the 100 people is  $3.05(20) = 61$ .  
When  $n = 20$ ,  $E(X) = 18.57$ . The total number of tests for the 100 people is  $18.57(5) = 92.9$ .  
Therefore to minimize the number of tests we must place them in groups of 4.