

University of California, Los Angeles
Department of Statistics

Statistics 13

Instructor: Nicolas Christou

Practice problems

Problem 1

Diseases I and II are common among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II , and 3% will contract both diseases.

- a. Find the probability that a randomly chosen person from this population will contract at least one disease?
- b. Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.
- c. Are the events “contracting disease I ” and “contracting disease II ” independent?

Problem 2

Part A:

Answer the following questions:

- a. You toss simultaneously 3 fair coins until all three show the same face. What is the probability that all three coins show the same face on the third attempt?
- b. Let events A, B . What probability does the expression $P(A) + P(B) - 2P(A \cap B)$ represent?
- c. You randomly select 2 cards without replacement from a standard 52-card deck that has 13 clubs (\clubsuit), 13 spades (\spadesuit), 13 diamonds (\diamondsuit), and 13 hearts (\heartsuit). What is the probability that both cards are of the same suit?

Part B:

Let events A, B . Show that

$$P(A|B) + P(A'|B) = 1$$

Problem 3

Part A:

Indicate (without computation) which list has the higher standard deviation:

- a. List A: 20, 20, 20, 20, 20
List B: 20, 20, 20, 20, 19
- b. List A: 20, 25, 25, 25, 30
List B: 15, 25, 25, 25, 35
- c. List A: 20, 20, 30, 40, 40
List B: 20, 25, 30, 35, 40
- d. List A: 1,1,1,2,2,2,3,3,3,4,4,4
List B: 1,1,1,1,2,2,2,2,3,3,3,3,4,4,4,4

Part B:

For a sample of size $n = 50$ you are given the following: $\sum_{i=1}^{50} x_i^2 = 100, \bar{x} = 0.8$. If you have enough information please compute the sample variance.

Problem 4

Answer the following questions:

- Five cards are selected without replacement from an ordinary 52-card deck. Find the probability that you obtain at least 1 clubs.
- For two events A, B it is given that $P(A) = 0.3, P(B) = 0.6, P(A \cap B) = 0.1$. Find the probability that none of these two events occurs.
- Two dice are thrown n times in succession. How large need n be so that the probability of at least one double six is at least $\frac{1}{2}$.
- Two dice are rolled and the sum of the two numbers is observed. Given that the sum is 10 what is the probability that the double five occurred?
- In the game of graps two dice are rolled and the sum is observed. If a player rolls a sum of 5 then he will have to roll the two dice again until the sum of 5 or the sum of 7 is observed. If the sum of 5 is observed before the sum of 7 the player wins. If the sum of 7 is observed before the sum of 5 the player loses. A player rolls the sum of 5 on the first trial. What is the probability that he wins on the 5th trial?

Problem 5**Part A:**

Bowl B_1 contains 3 white and 9 green chips. Bowl B_2 contains 8 white and 4 green chips. Bowl B_3 contains 10 white and 2 green chips. A bowl is selected at random (with equal probabilities), and one chip is selected.

- Compute the probability of selecting a white chip.
- Suppose a bowl was selected and handed to you without your being told which bowl you hold. If you select a white chip from this bowl, what is the conditional probability that you were handed bowl B_3 ?

Part B:

Two events A and B are such that $P(A) = 0.2, P(B) = 0.3$ and $P(A \cup B) = 0.4$.

- Find $P(A \cap B)$.
- Find $P(A' \cup B')$.
- Find $P(A'|B)$.

Problem 6**Part A:**

For a gambling game if a player bets \$1 his expected profit is $E(X) = -\$0.053$ with variance $Var(X) = \$33.21$. What is his expected profit and variance of the following two situations:

- The player will bet \$40 and play the game once.
- The player will play the game 40 times and will bet each time \$1.

Part B:

In a gambling game a player who draws a jack or a queen is paid \$15 and \$5 for drawing a king or an ace from an ordinary deck of fifty-two playing cards. A player who draws any other card pays \$4. Let X be the player's profit. Find the expected value and variance of X .

Problem 7

In a bolt factory machines A, B, C manufacture, respectively, 25, 35, and 40 percent of the total. It is known that of their output 5, 4, and 2 per cent are defective bolts. The bolts are shipped to warehouses and suppose that a bolt is drawn at random from a certain warehouse and is found defective. What is the probability it was *not* manufactured by machine A ?