University of California, Los Angeles
Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Practice problems

Problem 1
Diseases I and II are common among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract diesease II, and 3% will contract both diseases.

a. Find the probability that a randomly chosen person from this population will contract at least one disease?
$P(I \cup II) = P(I) + P(II) - P(I \cap II) = 0.10 + 0.15 - 0.03 = 0.22$.

b. Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.
$P(I \cap II / I \cup II) = \frac{P((I \cap II) \cap (I \cup II))}{P(I \cup II)} = \frac{P(I \cup II / I \cap II) P(I \cap II)}{P(I \cup II)} = \frac{1(0.03)}{0.22} = \frac{3}{22}$.

c. Are the events "contracting disease I" and "contracting disease II" independent?
No. $P(I \cap II) = 0.03 \neq P(I)P(II) = 0.10(0.15) = 0.015$.

Problem 2
Assume that 4% of the population has type AB blood.

a. Suppose that 10 blood donors will be checked. What is the probability that at least one of them has type AB blood? Assume that the donors are independent of each other.
$P(\text{at least one}) = 1 - P(\text{none}) = 1 - 0.96^{10} = 0.3352$.

b. How many blood donors must be checked in order to find at least one AB type donor with probability at least 80%? Assume that the donors are independent of each other.
$1 - 0.96^n = 0.80 \Rightarrow n = \frac{log(0.20)}{log(0.96)} = 39.4$. Therefore at least 40 donors.

Problem 3
There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and the third is a biased coin that comes heads 75% of the time.

a. A coin is randomly selected and flipped. What is the probability that it shows heads?

b. When one coin is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

c. The same coin (from part (b)) is flipped again and it shows heads. What is the probability that it is the fair coin? *Hint:* It is given that the same coin when flipped showed heads and then again heads.

Define the events:
$A = \{\text{two-headed coin was selected}\}$, $F = \{\text{fair coin was selected}\}$, $B = \{\text{biased coin was selected}\}$,

a. $P(H) = P(H \cap A) + P(H \cap F) + P(H \cap B) = P(H/A)P(A) + P(H/F)P(F) + P(H/B)P(B) = 1\frac{1}{3} + \frac{1}{2}\frac{1}{3} + \frac{3}{4}\frac{1}{3} = \frac{3}{4}$.

b. $P(A/H) = \frac{P(A \cap H)}{P(H)} = \frac{P(H/A)P(A)}{P(H/A)P(A) + P(H/F)P(F) + P(H/B)P(B)} = \frac{1\frac{1}{3}}{1\frac{1}{3} + \frac{1}{2}\frac{1}{3} + \frac{3}{4}\frac{1}{3}} = \frac{4}{9}$.

c. $P(F/HH) = \frac{P(F \cap HH)}{P(HH)} = \frac{P(HH/F)P(F)}{P(HH/F)P(F) + P(HH/B)P(B) + P(HH/A)P(A)} = \frac{\frac{1}{4}\frac{1}{3}}{\frac{1}{4}\frac{1}{3} + \frac{9}{16}\frac{1}{3} + 1\frac{1}{3}} = \frac{4}{29}$.