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Department of Statistics

Statistics 13 — Instructor: Nicolas Christou

Practice problems

Problem 1
Let A and B be events having positive probabilities. State whether each of the following statements is (i) true, (ii) false, (iii) could happen.

a. If A and B are mutually exclusive, then they are independent.
Ans: FALSE.

b. If A and B are independent then they are not mutually exclusive.
Ans: TRUE.

c. $P(A) = P(B) = 0.6$, and A and B are mutually exclusive.
Ans: FALSE.

d. $P(A) = P(B) = 0.6$, and A and B are independent.
Ans: COULD HAPPEN.

e. $P(A) = 0.6, P(B) = 0.3, P(A \cap B) = 0.1$ and $P(A' \cap B') = 0.3$.
Ans: FALSE.

Problem 2
An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.40, whereas this probability decreases to 0.20 for a non-accident-prone person. Assume that 30% of the population is accident prone.

a. What is the probability that that a new policyholder will have an accident within a year of purchasing a policy?
Ans: Let $A = \{person\ will\ have\ an\ accident\}$, $B = \{person\ is\ accident - prone\}$ $C = \{person\ is\ not\ accident - prone\}$.
$P(A) = P(A \cap B) + P(A \cap C) = P(A|B)P(B) + P(A|C)P(C) = (0.40)(0.3) + (0.20)(0.70) = 0.26$.

b. What is the probability that that a new policyholder will not have an accident within a year of purchasing a policy?
Ans: $P(A') = 1 - P(A) = 1 - 0.26 = 0.74$.

c. Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is an accident prone?
Ans: $P(B|A) = \frac{P(A \cap B)}{P(B)} = \frac{(0.40)(0.3)}{(0.40)(0.3)+(0.20)(0.70)} = 0.4615$.

Problem 3
Answer the following questions:

a. The probability of receiving a one-pair poker hand is 0.42. What is the probability that a player will receive at least one one-pair hand in 10 games?
Ans: This is binomial with $n = 10, p = 0.42$.
$P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{10}{0} 0.42^0 0.58^{10} = 1 - 0.58^{10} = 0.9957$.

b. Compute the expected value and standard deviation of the *maximum* number when two dice are rolled. You will have to find the distribution of the maximum (X) first (i.e. complete the table below).

X	$P(X)$
1	$\frac{1}{36}$
2	$\frac{3}{36}$
3	$\frac{5}{36}$
4	$\frac{7}{36}$
5	$\frac{9}{36}$
6	$\frac{11}{36}$

$E(X) = 1 \times \frac{1}{36} + \ldots + 6 \times \frac{11}{36} = 4.47$.
$var(X) = 1^2 \times \frac{1}{36} + \ldots + 6^2 \times \frac{11}{36} - 4.47^2 = 1.99$.
$sd(X) = \sqrt{1.99} = 1.41$.

c. A random variable X has $E(X) = 10$ and $SD(X) = 5$. Find $E(2X^2 + X)$.
Ans: $E(2X^2 + X) = 2EX^2 + EX = 2(\sigma^2 + \mu^2) + \mu = 2(25 + 10^2) + 10 = 260$.

d. Two players A, B compete for a prize by drawing a card with replacement from a standard 52-card deck until one receives an ace. What is the probability that player A wins? Note: player A begins the game.
Ans: $P(A) = \frac{4}{52} + (\frac{48}{52})^2 \frac{4}{52} + (\frac{48}{52})^4 \frac{4}{52} + \ldots = \frac{\frac{4}{52}}{1-(\frac{48}{52})^2} = 0.52$.

Problem 4
Answer the following questions:

a. If $E(X) = 1$ and $Var(X) = 5$ compute $Var(4 + 3X)$.
Ans: $Var(4 + 3X) = 3^2 Var(X) = 45$.

b. A box contains 4 green and 5 blue marbles. Two marbles are withdrawn randomly without replacement. If they are the same color, then you win \$1.10. If they are the different colors, then you lose \$1.10. Calculate the expected value of the amount you win.

Ans: Here is the probability distribution:

Color	$X(\$)$	$P(X)$
GG	1.10	$\frac{4}{9}\frac{3}{8} = \frac{12}{72}$
BB	1.10	$\frac{5}{9}\frac{4}{8} = \frac{20}{72}$
GB	-1.10	$\frac{4}{9}\frac{5}{8} = \frac{20}{72}$
BG	-1.10	$\frac{5}{9}\frac{4}{8} = \frac{20}{72}$

$E(X) = 1.10 \times \frac{12}{72} + 1.10 \times \frac{20}{72} - 1.10 \times \frac{20}{72} - 1.10 \times \frac{20}{72} = -0.12$.

c. An insurance company must pay a policyholder an amount A if some event E occurs within a year. Suppose the event E will occur with probability p. What should the company charge the customer in order that its expected profit will be 10% of A?
Ans: Let Q the amount the insurance company should charge the policyholder. The probability distribution will be:

	Profit X	$P(X)$
Event E	$-A + Q$	p
Event E'	Q	$1 - p$

We want: $(-A + Q)p + Q(1 - p) = 0.10A$. Solve for Q.

d. An electronic system consists of two components which are connected in series. Each component has its backup component and the system fails if a component and its backup component fails. All the components are independent of each other and they fail with probability 0.10. What is the probability that the system functions?

Ans: $(1 - 0.1^2) \times (1 - 0.1^2) = 0.9801$.