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Department of Statistics

Statistics 13 **Instructor: Nicolas Christou**

Practice problems

Problem 1

The game of craps is described below: A player rolls two dice, and the sum of the two numbers that appear is recorded. If the sum on the first roll is 7 or 11, the player wins immediately and the game stops. If on the first attempt a sum of 2, or 3, or 12 is scored, the player loses the game immediately. If the sum on the first roll is 4, 5, 6, 8, 9, or 10, then the two dice are rolled again until either the sum of 7 is scored or the original sum is scored. If the original sum is obtained a second time before 7, then the player wins. If the sum of 7 is obtained before the original sum is obtained a second time, then the player loses.

a. What is the probability that the player wins on his first attempt?

$$P(7 \text{ or } 11 \text{ on first}) = P(7) + P(11) = \frac{8}{36}.$$

b. Consider the following two events:
$A = \{$the player rolls a 6 on his first attempt and then another 6 before 7$\}$.
$B = \{$the player rolls a 5 on his first attempt and then another 5 before 7$\}$.
Are events A, B independent or mutually exclusive?
Mutually exclusive.

c. What is the probability that the player rolls 6 on his first attempt and then wins? This means that he first rolls a 6 and then will roll another 6 before 7.

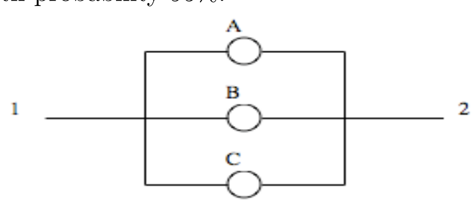
$$P(6)P(6 \text{ before } 7 \text{ thereafter}) = \frac{5}{36}\frac{\frac{5}{36}}{\frac{5}{36}+\frac{6}{35}} = \frac{5}{36}\frac{5}{11} = 0.0631.$$

d. It turns out that the probability of winning the game of craps is 0.49293. Describe how you can verify this probability. No calculations are needed, just the expressions. Hint: List all the possible ways that the player can win.

$$P(W) = P(7 \text{ or } 11 \text{ on first}) + \sum_{x=4,5,6,8,9,10} P(x \text{ on first})P(x \text{ before } 7 \text{ thereafter}).$$

Problem 2

On the figure below you see an electric system with three components that operate independently each one with probability 95%.



a. Find the probability that current will flow from point 1 to point 2.

$$P(F) = 1 - 0.05^3 = 0.999875.$$

b. Given that current flowed from point 1 to point 2, what is the probability that component A functioned?

$$P(A|F) = \frac{P(A \cap F)}{P(F)} = \frac{P(F|A)P(A)}{P(F)} = \frac{P(A)}{P(F)} = \frac{0.95}{0.999875} = 0.95012.$$

c. Let X be the number of paths that allow current to flow from point 1 to point 2. Construct the probability distribution of X. Note: $AB'C'$ is one path because A functions while B and C do not function. Also, X takes the value 0, 1, 2, 3. If $x = 0$ then there is no path that allows current to flow, while if $x = 3$ there are three paths that allow current to flow, etc.

X	$P(X)$
0	$0.05^3 = 0.000125$
1	$3(0.95)(0.05)^2 = 0.007125$
2	$3(0.95)^2(0.05) = 0.135375$
3	$0.95^3 = 0.857375$

Problem 3

Answer the following questions:

a. The odds of an event A is defined by the ratio $\frac{P(A)}{1-P(A)}$. For example is $P(A) = \frac{2}{3}$ then $1 - P(A) = \frac{1}{3}$, and therefore the odds ratio is 2. Sometimes we say that the odds are "2 to 1". The odds that the Los Angeles Lakers will win their next game is "4 to 1". What is the probablity that the Los Angeles Lakers will win the next game?

$$\frac{P(A)}{1 - P(A)} = 4 \Rightarrow P(A) = \frac{4}{5}.$$

b. The probability that a patient has disease I is 15%, disease II is 5% and both 3%. Let X be the number of diseases that this patient has. Construct the probability distribution of X and compute its mean and standard deviation.

X	$P(X)$
0	0.83
1	0.14
2	0.03

$E(X) = 0(0.83) + 1(0.14) + 2(0.03) = 0.2.$

$SD(X) = \sqrt{0^2(0.83) + 1^2(0.14) + 2^2(0.03) - 0.2^2} = \sqrt{0.22} = 0.4690.$