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Statistics 13 — Instructor: Nicolas Christou

Practice problems - Solutions

Problem 1
Diseases I and II are common among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract diesease II, and 3% will contract both diseases.

a. Find the probability that a randomly chosen person from this population will contract at least one disease?
$P(I \cup II) = P(I) + P(II) - P(I \cap II) = 0.10 + 0.15 - 0.03 = 0.22$.

b. Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.
$P(I \cap II / I \cup II) = \frac{P((I \cap II) \cap (I \cup II))}{P(I \cup II)} = \frac{P(I \cup II / I \cap II) P(I \cap II)}{P(I \cup II)} = \frac{1(0.03)}{0.22} = \frac{3}{22}$.

c. Are the events “contracting disease I” and “contracting disease II” independent?
No. $P(I \cap II) = 0.03 \neq P(I)P(II) = 0.10(0.15) = 0.015$.

Problem 2
Part A:

a. This is geometric: $P(X = 3) = (\frac{3}{4})^2 \frac{1}{4}$.

b. It represents the probability of A or B but NOT both.

c. This is the probability that two clubs, or two spades or two hears, or two diamonds are selected. Let A be this event:
$P(A) = 4 \frac{13}{52} \frac{12}{51}$.

Part B:
$P(A|B) + P(A'|B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$.

Problem 4

a. $1 - \frac{39}{52} \frac{38}{51} \frac{37}{50} \frac{36}{49} \frac{35}{48}$.

b. $PA' \cap B') = 1 - P(A \cup B) = 1 - (0.3 + 0.6 - 0.1) = 0.2$.

c. $n = \frac{log(0.5)}{log(\frac{35}{36})} \approx 25$.

d. Let A be the event that the double five pair occurred, and B the event that the sum is ten.
$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{1(\frac{1}{36})}{\frac{3}{36}} = \frac{1}{3}$.

d. This is the sequence of the following events:
five on first
no five or seven on second
no five or seven on third
no five or seven on fourth
five on fifth

and the probability is equal to: $\frac{4}{36} \times \frac{26}{36} \times \frac{26}{36} \times \frac{26}{36} \times \frac{4}{36}$.

Problem 5
Part A:

a. $$P(W) = P(W \cap B_1) + P(W \cap B_2) + P(W \cap B_3) = \frac{3}{12}\frac{1}{3} + \frac{8}{12}\frac{1}{3} + \frac{10}{12}\frac{1}{3} = 0.5833.$$

b. $$P(B_3|W) = \frac{P(W \cap B_3)}{P(W)} = \frac{P(W|B_3)P(B_3)}{P(W)} = \frac{\frac{10}{12}\frac{1}{3}}{0.5833} = 0.4762.$$

Part B:

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.2 + 0.3 - 0.4 = 0.1$.

b. $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.1 = 0.9$.

c. From the Venn diagram we find that $P(A' \cap B) = 0.2$. Therefore: $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.2}{0.3} = \frac{2}{3}$.

Problem 6
Part A:

a. $E(40X) = 40(-0.053) = -2.12$ and $var(40X) = 40^2(33.21) = 53136$.

b. $E(X_1 + \ldots + X_{40}) = 40(-0.053) = -2.12$, and $var(X_1 + \ldots + X_{40}) = 40(33.21) = 1328.4$.

Part B:
Let X be the player's profit:

Card	$X(\$)$	$P(X)$
Jack	15	$\frac{4}{52}$
Queen	15	$\frac{4}{52}$
King	5	$\frac{4}{52}$
Ace	5	$\frac{4}{52}$
Else	-4	$\frac{36}{52}$

Therefore the expected gain is:
$E(X) = 15\frac{4}{52} + 15\frac{4}{52} + 5\frac{4}{52} + 5\frac{4}{52} - 4\frac{36}{52} = \$\frac{16}{52}$.

Problem 7
It is given $P(A) = 0.25, P(B) = 0.35, P(C) = 0.40$, and $P(D|A) = 0.05, P(D|B) = 0.04, P(D|C) = 0.02$. Let's find the probability that it was manufactured by machine A given it was found defective.

$$P(A|D) = \frac{P(D \cap A)}{P(D)} = \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)} = \frac{0.05 \times 0.25}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.40} = 0.3623.$$

Therefore the probability that it was not manufactured by machine A is: $P(A'|D) = 1 - P(A|D) = 1 - 0.3623 = 0.6377$.