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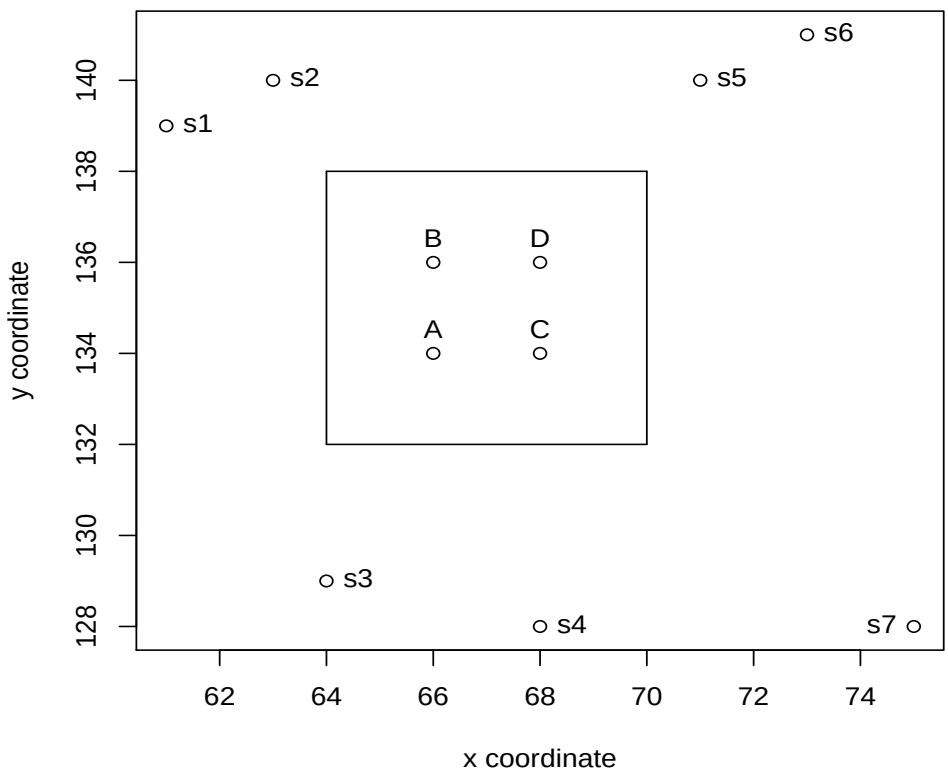
Statistics C173/C273 **Instructor: Nicolas Christou**

Block kriging exampe

We will use the 7-point data from earlier lectures. Here they are:

s_i	x	y	$z(s_i)$
s_1	61	139	477
s_2	63	140	696
s_3	64	129	227
s_4	68	128	646
s_5	71	140	606
s_6	73	141	791
s_7	75	128	783

Here is the $x - y$ plot:



For these data, let's assume that we use the exponential semivariogram model with parameters $c_0 = 0, c_1 = 10, \alpha = 3.33$, $\gamma(h) = 10(1 - e^{-\frac{h}{3.33}})$. The corresponding covariance function is:

$$C(h) = \begin{cases} 10 & h = 0 \\ 10e^{-\frac{h}{3.33}} & h > 0 \end{cases}$$

Suppose we want to estimate the average of the block defined by the coordinates shown on the figure of page 1: $(64, 132), (64, 138), (70, 132), (70, 138)$ (see figure on page 1).

One way to do this is to "krige" many points inside the block and at the end average these estimates. Let's use only 4 points inside the block defined by the following coordinates:
$A(66, 134), B(66, 136), C(68, 134), D(68, 136)$.

Predicting point A:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.196 \\ 1.334 \\ 1.985 \\ 1.497 \\ 0.958 \\ 0.512 \\ 0.388 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.130 \\ 0.146 \\ 0.240 \\ 0.164 \\ 0.136 \\ 0.072 \\ 0.112 \\ -1.077 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_A) = 0.130(477) + 0.146(696) + 0.240(227) + 0.164(646) + 0.136(606) + 0.072(791) + 0.112(783) \Rightarrow$
$\hat{z}(s_A) = 551.036$.

Predicting point B:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.736 \\ 2.228 \\ 1.123 \\ 0.841 \\ 1.462 \\ 0.755 \\ 0.269 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.143 \\ 0.227 \\ 0.161 \\ 0.117 \\ 0.181 \\ 0.068 \\ 0.104 \\ -1.050 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_B) = 0.143(477) + 0.227(696) + 0.161(227) + 0.117(646) + 0.181(606) + 0.068(791) + 0.104(783) \Rightarrow$
$\hat{z}(s_B) = 582.733$.

Predicting point C:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0.755 \\ 0.958 \\ 1.462 \\ 1.650 \\ 1.334 \\ 0.755 \\ 0.627 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.100 \\ 0.125 \\ 0.183 \\ 0.197 \\ 0.176 \\ 0.081 \\ 0.137 \\ -1.120 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_C) = 0.100(477) + 0.125(696) + 0.183(227) + 0.197(646) + 0.176(606) + 0.081(791) + 0.137(783) \Rightarrow$
$\hat{z}(s_C) = 582.293.$

Predicting point D:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.016 \\ 1.462 \\ 0.888 \\ 0.905 \\ 2.228 \\ 1.196 \\ 0.411 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.101 \\ 0.168 \\ 0.139 \\ 0.132 \\ 0.263 \\ 0.078 \\ 0.120 \\ -1.090 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_D) = 0.101(477) + 0.168(696) + 0.139(227) + 0.132(646) + 0.263(606) + 0.078(791) + 0.120(783) \Rightarrow$
$\hat{z}(s_D) = 596.347.$

Note: All the covariances above are shown on the next page.

We can now average these 4 estimates to get an estimate of the mean of the block:

$$\hat{Z}_{BLOCK} = \frac{551.036 + 582.733 + 582.293 + 596.347}{4} = 578.102.$$

Comment: As we observe we need to run kriging 4 times. And if there are many points within the block we will have to run kriging many times. This may be computationally expensive. The other way to do this is to use block kriging. With block kriging you need only to run kriging once. Here are the details: The covariance matrix $\boldsymbol{\Sigma}$ which constructed using the observed data points will be the same as shown above. What changes, are the entries of the vector $\boldsymbol{c}$. Each entry of this vector is the average of the covariances between each observed data point with every point in the block.

Here is the distance matrix. This is an 11×11 matrix (4 points to be estimated plus the 7 observed points).

$$\text{Distance} = \begin{pmatrix} & A & B & C & D & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ A & 0.000 & 2.000 & 2.000 & 2.828 & 7.071 & 6.708 & 5.385 & 6.325 & 7.810 & 9.899 & 10.817 \\ B & 2.000 & 0.000 & 2.828 & 2.000 & 5.831 & 5.000 & 7.280 & 8.246 & 6.403 & 8.602 & 12.042 \\ C & 2.000 & 2.828 & 0.000 & 2.000 & 8.602 & 7.810 & 6.403 & 6.000 & 6.708 & 8.602 & 9.220 \\ D & 2.828 & 2.000 & 2.000 & 0.000 & 7.616 & 6.403 & 8.062 & 8.000 & 5.000 & 7.071 & 10.630 \\ s_1 & 7.071 & 5.831 & 8.602 & 7.616 & 0.000 & 2.236 & 10.440 & 13.038 & 710.050 & 12.166 & 17.804 \\ s_2 & 6.708 & 5.000 & 7.810 & 6.403 & 2.236 & 0.000 & 11.045 & 13.000 & 8.000 & 10.050 & 16.971 \\ s_3 & 5.385 & 7.280 & 6.403 & 8.062 & 10.440 & 11.045 & 0.000 & 4.123 & 13.038 & 15.000 & 11.045 \\ s_4 & 6.325 & 8.246 & 6.000 & 8.000 & 13.038 & 13.000 & 4.123 & 0.000 & 12.369 & 13.928 & 7.000 \\ s_5 & 7.810 & 6.403 & 6.708 & 5.000 & 10.050 & 8.000 & 13.038 & 12.369 & 0.000 & 2.236 & 12.649 \\ s_6 & 9.899 & 8.602 & 8.602 & 7.071 & 12.166 & 10.050 & 15.000 & 13.928 & 2.236 & 0.000 & 653.547 \\ s_7 & 10.817 & 12.042 & 9.220 & 10.630 & 17.804 & 16.971 & 11.045 & 7.000 & 12.649 & 13.153 & 0.000 \end{pmatrix}.$$

And here are the covariances between all the points:

$$\text{Covariance} = \begin{pmatrix} & A & B & C & D & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ A & 10.000 & 5.485 & 5.485 & 4.277 & 1.196 & 1.334 & 1.985 & 1.497 & 0.958 & 0.512 & 0.388 \\ B & 5.485 & 10.000 & 4.277 & 5.485 & 1.736 & 2.228 & 1.123 & 0.841 & 1.462 & 0.755 & 0.269 \\ C & 5.485 & 4.277 & 10.000 & 5.485 & 0.755 & 0.958 & 1.462 & 1.650 & 1.334 & 0.755 & 0.627 \\ D & 4.277 & 5.485 & 5.485 & 10.000 & 1.016 & 1.462 & 0.888 & 0.905 & 2.228 & 1.196 & 0.411 \\ s_1 & 1.196 & 1.736 & 0.755 & 1.016 & 10.000 & 5.109 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 \\ s_2 & 1.334 & 2.228 & 0.958 & 1.462 & 5.109 & 10.000 & 0.363 & 0.202 & 0.905 & 0.489 & 0.061 \\ s_3 & 1.985 & 1.123 & 1.462 & 0.888 & 0.435 & 0.363 & 10.000 & 2.899 & 0.199 & 0.111 & 0.363 \\ s_4 & 1.497 & 0.841 & 1.650 & 0.905 & 0.199 & 0.202 & 2.899 & 10.000 & 0.244 & 0.153 & 1.222 \\ s_5 & 0.958 & 1.462 & 1.334 & 2.228 & 0.489 & 0.905 & 0.199 & 0.244 & 10.000 & 5.109 & 0.224 \\ s_6 & 0.512 & 0.755 & 0.755 & 1.196 & 0.259 & 0.489 & 0.111 & 0.153 & 5.109 & 10.000 & 0.000 \\ s_7 & 0.388 & 0.269 & 0.627 & 0.411 & 0.048 & 0.061 & 0.363 & 1.222 & 0.224 & 0.193 & 10.000 \end{pmatrix}.$$

The block kriging system:

$$\begin{pmatrix} c(s_1,s_1) & c(s_1,s_2) & c(s_1,s_3) & \cdots & c(s_1,s_n) & 1 \\ c(s_2,s_1) & c(s_2,s_2) & c(s_2,s_3) & \cdots & c(s_2,s_n) & 1 \\ \cdots & \cdots & \ddots & \cdots & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \cdots & 1 \\ c(s_n,s_1) & c(s_n,s_2) & c(s_n,s_3) & \cdots & c(s_n,s_n) & 1 \\ 1 & 1 & \cdots & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_n \\ -\lambda \end{pmatrix} = \begin{pmatrix} c(s_1,A) \\ c(s_2,A) \\ \vdots \\ \vdots \\ c(s_n,A) \\ 1 \end{pmatrix}$$

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.176 \\ 1.495 \\ 1.365 \\ 1.223 \\ 1.495 \\ 0.805 \\ 0.424 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.118 \\ 0.166 \\ 0.181 \\ 0.152 \\ 0.189 \\ 0.075 \\ 0.118 \\ -1.084 \end{pmatrix}.$$

Therefore,
$\hat{z}_{BLOCK} = 0.118(477)+0.166(696)+0.181(227)+0.152(646)+0.189(606)+0.075(791)+0.118(783) \Rightarrow$
$\hat{z}_{BLOCK} = 578.102.$

Note: Each element of the vector $\boldsymbol{c}$ is the average of the covariances between each observed point with each point in the block. For example the first entry 1.176 was computed as follows (from the covariance matrix on page 4):

$$\frac{1.196 + 1.736 + 0.755 + 1.016}{4} = 1.176$$

We can also observe that the weights of the block kriging system are the averages of the weights of the four ordinary kriging systems from pages 2-3.

Spatial data

"A major pleasure in working with spatial data is their visualization. Maps are amongst the most compelling graphics, because the space they map is the space we think we live in, and maps can show things we cannot see otherwise".

Roger Bivand, Edzer Pebesma, Virgilio Gomez-Rubio, Applied Spatial Data Analysis with R, Use R!, Springer 2008.