University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Homework 9

Exercise 1
Access the *Maas* river data at:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/
     soil1.txt", header=TRUE)
```

a. Perform cokriging predictions on a grid (use `by=50`), using `log(lead)` as the target variable and co-located variables `log(cadmium)`, `log(copper)`, and `log(zinc)`.

b. Use cross-validation to compare ordinary kriging with cokriging of question (a). Note: Cross-validation for ordinary kriging can be done with the `krige.cv` function, while cross-validation for cokriging can be done using the `gstat.cv` function.

Exercise 2
Read the following data in `R`:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/jura.txt",  header=TRUE)
```

Use `log(Pb)` as the target variable. Answer the following questions:

a. Create a `gstat` object using the target variable. Compute the sample omnidirectional variogram and fit a model to it.

b. Create a dense grid for kriging predictions (use `by=0.05`).

c. Construct the raster map using the back-transformed values $\check{Z}(s_0)$. Note: See handout #58.

d. Assign `NA` values to the region outside the observed points. Finally add contours to the map. Note: See handout #58.

Exercise 3
Consider the data shown on the rectangle of the previous page. Our goal is to estimate the average of the variable $Z(s)$ of the rectangle `ABCD`. The rectangle is descritized by the points `a,b,c,d,e,f`, and the observed data points are $z(s_1), z(s_2), z(s_3), z(s_4), z(s_5)$ (see the coordinates and figure below). Assume that the process Z is second order stationary with variogram function:

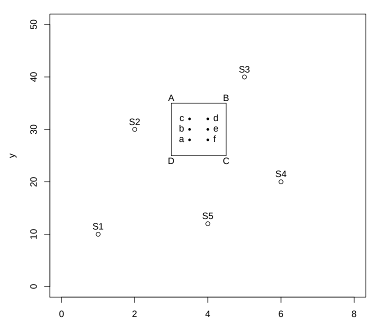
$$\gamma(h) = \begin{cases} 0 & h = 0 \\ 3(1 - e^{-\frac{h}{5}}) & h > 0 \end{cases}$$

a. Write the system of the kriging equations in terms of the variogram that will give you the weights for the estimation of the average of the rectangle `ABCD`. Explain what each part of the equations represent.

b. Compute the covariance between the observed data point s_1 and point `a` of the rectangle `ABCD`.

c. Write the expression that computes the covariance between the observed data point s_1 and the average of the rectangle `ABCD`.

Coordinates of the observed data points s_1, s_2, s_3, s_4, s_5 and the points a, b, c, d, e, f:

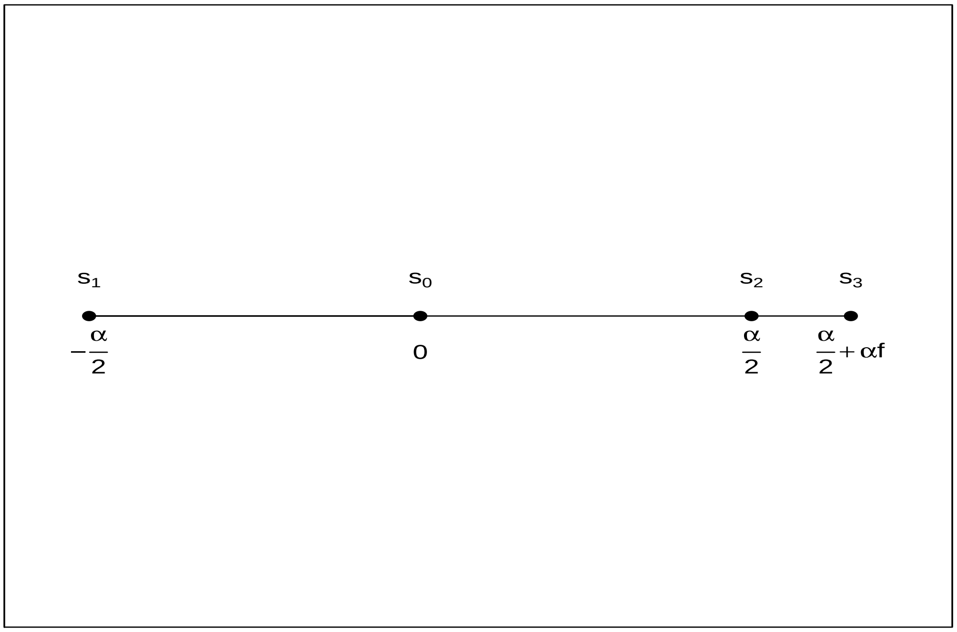


Point	x	y
s_1	1.0	10
s_2	2.0	30
s_3	5.0	40
s_4	6.0	20
s_5	4.0	12
a	3.5	28
b	3.5	30
c	3.5	32
d	4.0	28
e	4.0	30
f	4.0	32

Exercise 4
Consider the covariance function given by

$$C(h) = \begin{cases} c_1(1 - \frac{h}{\alpha}), & h \leq \alpha \\ 0, & h > \alpha \end{cases}$$

Suppose we want to predict point s_0 at $s_0 = 0$ using points $s_1 = -\frac{\alpha}{2}$, $s_2 = \frac{\alpha}{2}$ and $s_3 = \frac{\alpha}{2} + \alpha f$, with $0 < f \leq 1$. Note that when $\frac{1}{2} < f \leq 1$, point s_3 is located beyond the range of s_0.



Show that when $0 < f \leq \frac{1}{2}$ the ordinary kriging weights are $w_1 = \frac{2}{4-f}$, $w_2 = \frac{3-f}{4-f}$, and $w_3 = -\frac{1}{4-f}$. See handout #70 for the case when $\frac{1}{2} < f \leq 1$.