Variograms can be classified based on their behavior at infinity and the origin.

a. Behavior at infinity:
   1. Bounded.
   2. Unbounded.

a. Behavior at the origin:
   1. Linear.
   2. Quadratic.
   3. Discontinuous.

Basic permissible variogram models

a. **Power function**
   \[ \gamma(h) = \alpha h^\omega, \text{ with } 0 < \omega < 2. \]
b. Exponential

\[ \gamma(h; \theta) = \begin{cases} 
0 & h = 0 \\
c_0 + c_1(1 - \exp(-\frac{h}{\alpha})) & h \neq 0
\end{cases} \]

\[ \theta = (c_0, c_1, \alpha)', \text{ where } c_0 \geq 0, \ c_1 \geq 0, \ \text{and } \alpha \geq 0. \]

The exponential semivariogram function approaches its sill asymptotically and therefore its range is not finite. For practical purposes, a practical range is used which is equal to the distance at which the semivariogram is equal to 95\% of the sill.
c. Spherical

\[
\gamma(h; \theta) = \begin{cases} 
0 & h = 0 \\
c_0 + c_1 \left( \frac{3}{2} \left( \frac{h}{\alpha} \right) - \frac{1}{2} \left( \frac{h}{\alpha} \right)^3 \right) & 0 < h \leq \alpha \\
c_0 + c_1 & h \geq \alpha 
\end{cases}
\]

\[
\theta = (c_0, c_1, \alpha)', \text{ where } c_0 \geq 0, \ c_1 \geq 0, \text{ and } \alpha \geq 0.
\]

The spherical semivariogram has an actual range equal to \( \alpha \). If \( h \geq \alpha \) then \( \gamma(h) = c_0 + c_1 \). It must be “forced” to equal \( c_0 + c_1 \) when \( h \geq \alpha \) - see next plots.
d. Gaussian

\[
\gamma(h; \theta) = \begin{cases} 
0 & h = 0 \\
c_0 + c_1(1 - exp(-\frac{h^2}{\alpha^2})) & h \neq 0
\end{cases}
\]

\[\theta = (c_0, c_1, \alpha)^t, \text{ where } c_0 \geq 0, c_1 \geq 0, \text{ and } \alpha \geq 0.\]

The Gaussian semivariogram approaches its sill asymptotically. A practical range is used which is equal to the distance at which the semivariogram is equal to 95% of the sill.

A modified function can be used with an extra parameter \(\omega\) replacing the exponent 2 in the Gaussian model:

\[
\gamma(h; \theta) = \begin{cases} 
0 & h = 0 \\
c_0 + c_1(1 - exp(-\frac{h^\omega}{\alpha^\omega})) & h \neq 0
\end{cases}
\]

e. Matérn function

Very flexible function, but complicate! It is a general form of few of the models discussed above.

\[
\gamma(h; \theta) = c_0 + c_1 \left[ 1 - \frac{1}{2^{\nu-1}\Gamma(\nu)} \left( \frac{h}{\alpha} \right)^\nu K_\nu \left( \frac{h}{\alpha} \right) \right]
\]

\(\Gamma\) is the gamma function, and \(K\) is the Bessel function.

f. Pure nugget effect model

\[\gamma(h) = c_0, \text{ for } h > 0.\]