University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Variograms - summary

Variograms can be classified based on their behavior at infinity and the origin.

a. Behavior at infinity:

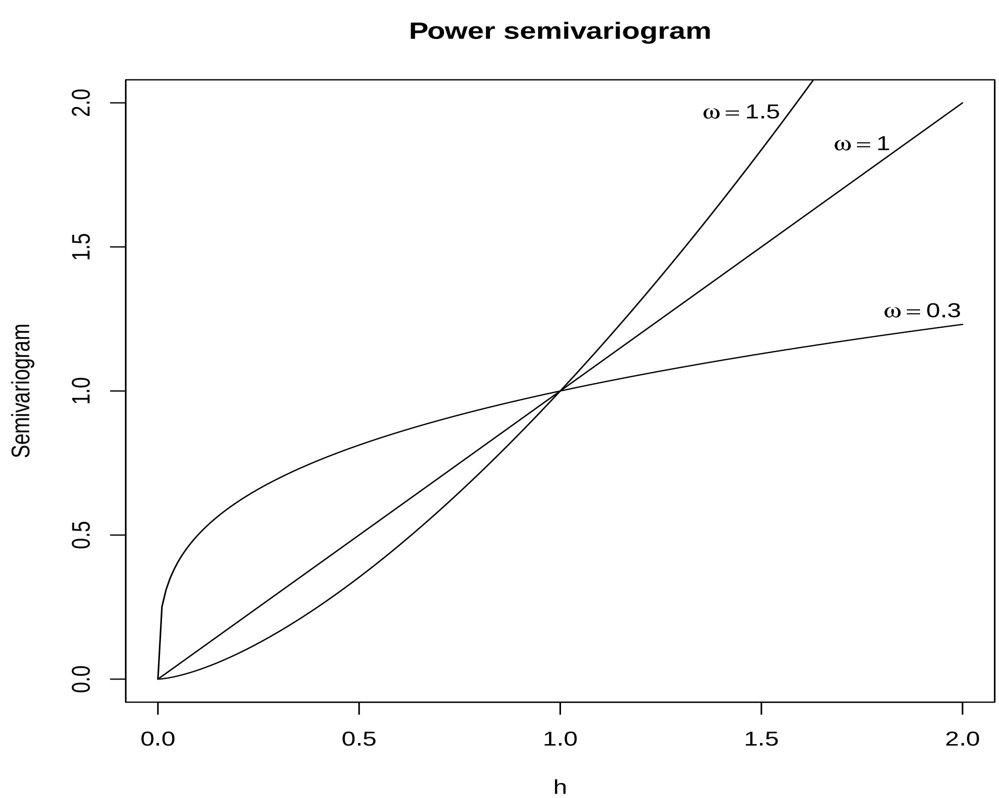
1. Bounded.
2. Unbounded.

a. Behavior at the origin:

1. Linear.
2. Quadratic.
3. Discontinuous.

Basic permissible variogram models

a. **Power function**
$\gamma(h) = \alpha h^{\omega}$, with $0 < \omega < 2$.

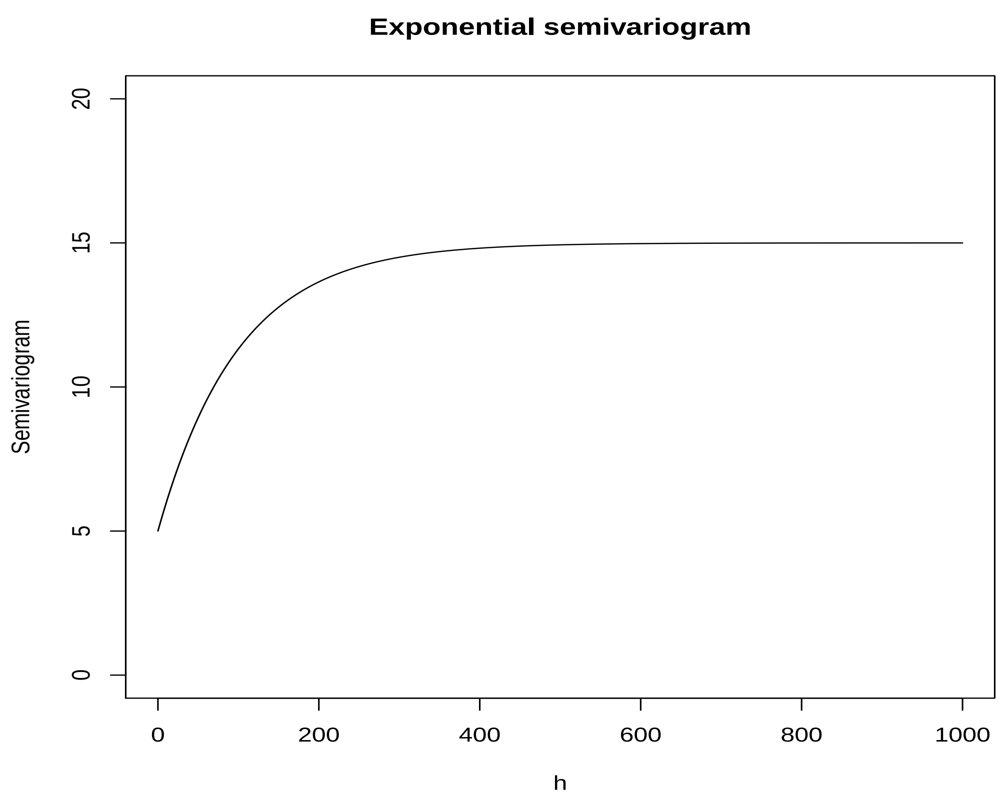


b. **Exponential**

$$\gamma(h;\boldsymbol{\theta}) = \begin{cases} 0 & h = 0 \\ c_0 + c_1(1 - exp(-\frac{h}{\alpha})) & h \neq 0 \end{cases}$$

$$\boldsymbol{\theta} = (c_0, c_1, \alpha)',\ \text{where } c_0 \geq 0,\ c_1 \geq 0,\ \text{and } \alpha \geq 0.$$

The exponential semivariogram function approaches its sill asymptotically and therefore its range is not finite. For practical purposes, a practical range is used which is equal to the distance at which the semivariogram is equal to 95% of the sill.

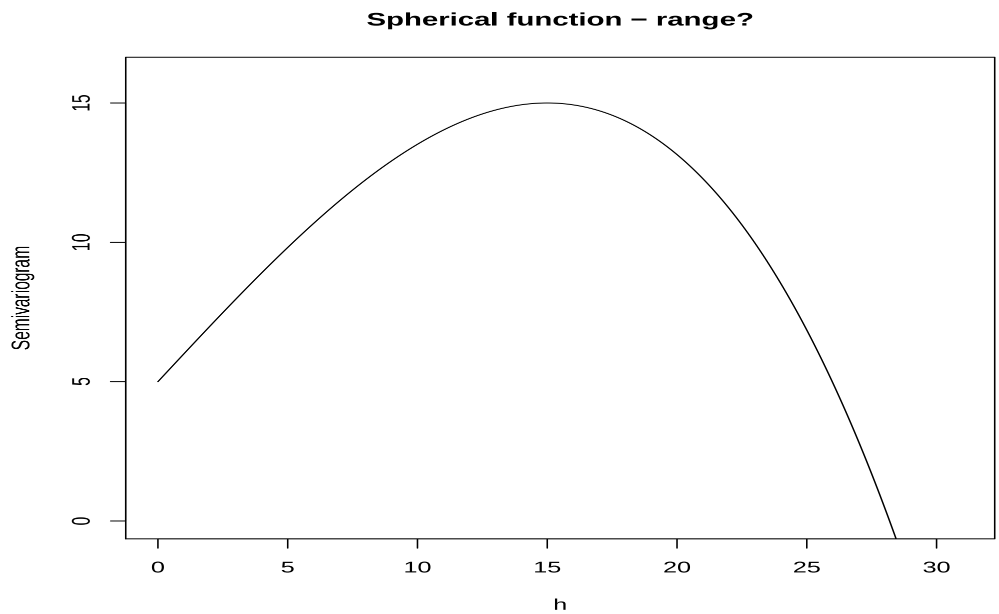


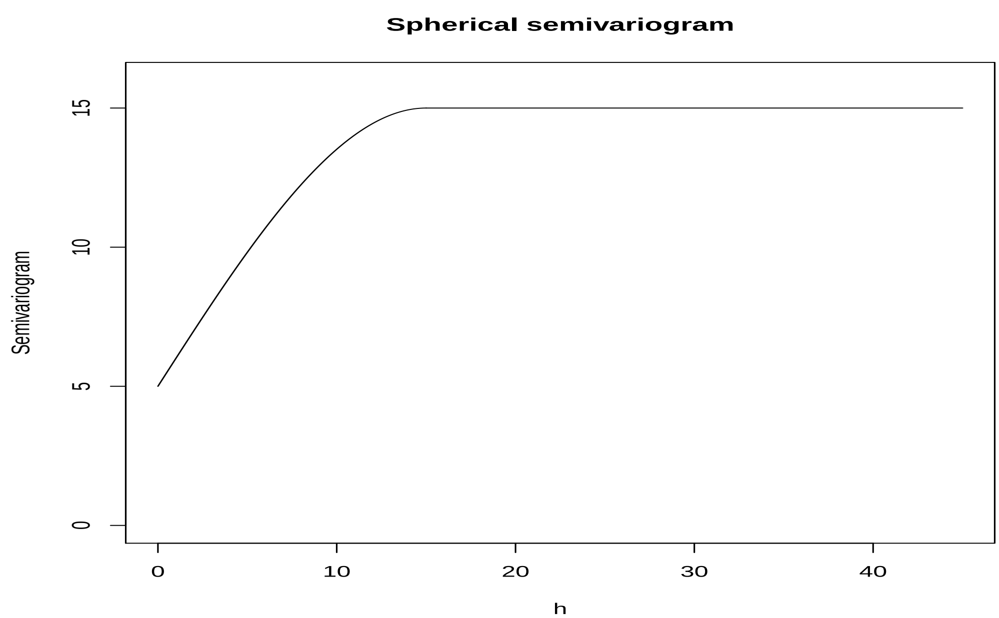
c. **Spherical**

$$\gamma(h;\boldsymbol{\theta}) = \begin{cases} 0 & h = 0 \\ c_0 + c_1(\frac{3}{2}(\frac{h}{\alpha}) - \frac{1}{2}(\frac{h}{\alpha})^3) & 0 < h \le \alpha \\ c_0 + c_1 & h \ge \alpha \end{cases}$$

$$\boldsymbol{\theta} = (c_0, c_1, \alpha)', \text{ where } c_0 \ge 0,\ c_1 \ge 0, \text{ and } \alpha \ge 0.$$

The spherical semivariogram has an actual range equal to α. If $h \ge \alpha$ then $\gamma(h) = c_0 + c_1$. It must be"forced" to equal $c_0 + c_1$ when $h \ge \alpha$ - see next plots.



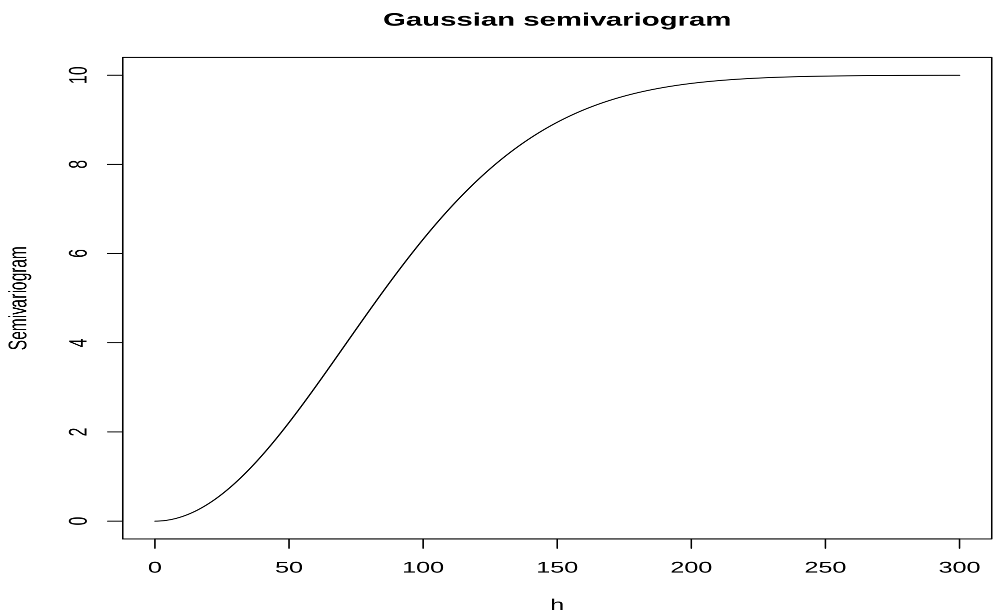


d. **Gaussian**

$$\gamma(h;\boldsymbol{\theta}) = \begin{cases} 0 & h = 0 \\ c_0 + c_1(1 - exp(-\frac{h^2}{\alpha^2})) & h \neq 0 \end{cases}$$

$$\boldsymbol{\theta} = (c_0, c_1, \alpha)', \text{ where } c_0 \geq 0, \; c_1 \geq 0, \text{ and } \alpha \geq 0.$$

The Gaussian semivariogram approaches its sill asymptotically. A practical range is used which is equal to the distance at which the semivariogram is equal to 95% of the sill.



A modified function can be used with an extra parameter ω replacing the exponent 2 in the Gaussian model:

$$\gamma(h;\boldsymbol{\theta}) = \begin{cases} 0 & h = 0 \\ c_0 + c_1(1 - exp(-\frac{h^\omega}{\alpha^\omega})) & h \neq 0 \end{cases}$$

e. **Matérn function**
Very flexible function, but complicate! It is a general form of few of the models discussed above.

$$\gamma(h;\boldsymbol{\theta}) = c_0 + c_1 \left[1 - \frac{1}{2^{\nu-1}\Gamma(\nu)} \left(\frac{h}{\alpha}\right)^{\nu} K_\nu \left(\frac{h}{\alpha}\right)\right]$$

Γ is the gamma function, and K is the Bessel function.

f. **Pure nugget effect model**
$\gamma(h) = c_0$, for $h > 0$.