SOME COMMENTS:

THE CO-KRIGING SYSTEM:

$$\begin{bmatrix} \underset{\sim}{\Sigma}_{11} & \underset{\sim}{\Sigma}_{12} & \underset{\sim}{1} & \underset{\sim}{0} \\ \underset{\sim}{\Sigma}_{21} & \underset{\sim}{\Sigma}_{22} & \underset{\sim}{0} & \underset{\sim}{1} \\ \underset{\sim}{1}' & \underset{\sim}{0}' & 0 & 0 \\ \underset{\sim}{0} & \underset{\sim}{1}' & 0 & 0 \end{bmatrix} \begin{bmatrix} \underset{\sim}{w}_1 \\ \underset{\sim}{w}_2 \\ -\lambda_1 \\ -\lambda_2 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{c}_{11} \\ \underset{\sim}{c}_{12} \\ 1 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{G} \qquad \underset{\sim}{w} \qquad \underset{\sim}{c}$$

$$\underset{\sim}{w} = \underset{\sim}{G}^{-1} \underset{\sim}{c}$$

$\underset{\sim}{\Sigma}_{12}$ MAY NOT BE THE SAME AS $\underset{\sim}{\Sigma}_{21}$.

THIS IS BECAUSE OF HOW CROSS-COVARIANCES ARE CALCULATED:

$$C_{12}(h) = E\left\{[Z_1(s) - \mu_1][Z_2(s+h) - \mu_2]\right\}$$

AND

$$\hat{C}_{12}(h) = \frac{1}{N(h)} \sum Z_1(s) \cdot Z_2(s+h) - \hat{\mu}_1 \cdot \hat{\mu}_2$$

OBVIOUSLY, $\hat{C}_{21} = \frac{1}{N(h)} \sum Z_2(s) \cdot Z_1(s+h) - \hat{\mu}_2 \cdot \hat{\mu}_1$

IS NOT NECESSARILY EQUAL TO $\hat{C}_{12}$.

EXAMPLE:

N

$z_1(s_1) = 10$ • $z_2(s_1) = 15$

$z_1(s_2) = 13$ • $z_2(s_2) = 16$

$z_1(s_3) = 12$ • $z_2(s_3) = 17$

$z_1(s_4) = 14$ • $z_2(s_4) = 19$

DISTANCE BETWEEN POINTS = 100 m

$z_1(s)$	$z_2(s+100)$
10	16
13	17
12	19

$$\hat{C}_{12}(100) = \frac{1}{3}\left[10\cdot 16 + 13\cdot 17 + 12\cdot 19\right] - \bar{z}_1(s)\cdot \bar{z}_2(s+100)$$

HOW ABOUT $\hat{C}_{21}(100)$:

$z_2(s)$	$z_1(s+100)$
15	13
16	12
17	14

$$\hat{C}_{21} = \frac{1}{3}\left[15\cdot 13 + 16\cdot 12 + 17\cdot 14\right] - \bar{z}_2(s)\cdot \bar{z}_1(s+100)$$

$$\therefore\ \hat{C}_{12} \neq \hat{C}_{21}$$

ASIDE:

$$C_{XY} = \frac{1}{n}\sum (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{n}\sum (x_i - \bar{x})\, y_i$$

$$= \frac{1}{n}\sum x_i y_i - \bar{x}\,\frac{\sum y_i}{n}$$

$$= \frac{1}{n}\sum x_i y_i - \bar{x}\,\bar{y}$$

THE CROSS-COVARIANCE IN GENERAL IS NOT EVEN FUNCTION:

$$C_{12}(h) \neq C_{12}(-h)$$

IN OTHER WORDS, THE CROSS-COVARIANCE IN ONE DIRECTION IS DIFFERENT FROM THE CROSS-COVARIANCE IN THE OPPOSITE DIRECTION.

THIS IS EASY TO OBSERVED FROM THE EXAMPLE ABOVE.

$$\hat{C}_{12}(-100) = \frac{1}{3}\left[14 \cdot 17 + 12 \cdot 16 + 13 \cdot 15\right] - \bar{Z}_1(s) \cdot \bar{Z}_2(s-100)$$

$Z_1(s)$	$Z_2(s-100)$
14	17
12	16
13	15

THEREFORE:

THE CO-KRIGING SYSTEM CANNOT BE FORMULATED IN TERMS OF THE VARIOGRAM UNLESS THE MATRIX $\underset{\sim}{\Sigma}_{12} = \underset{\sim}{\Sigma}_{21}$.

RECALL:

$$\hat{\gamma}_{12}(h) = \frac{1}{2N(h)} \sum \left[Z_1(s) - Z_1(s+h)\right]\left[Z_2(s) - Z_2(s+h)\right]$$

ANOTHER DEFINITION OF THE CROSS-VARIOGRAM IS THE FOLLOWING:

$$2\gamma_{12}(h) = E\left[Z_1(s) - Z_2(s+h)\right]^2$$

WITH ESTIMATOR

$$2\hat{\gamma}_{12}(h) = \frac{1}{N(h)} \sum \left[Z_1(s) - Z_2(s+h)\right]^2$$

ADVANTAGES:

1. IT CANNOT BECOME NEGATIVE
2. TARGET VARIABLE CAN BE UNDERSAMPLED
3. NUGGET EFFECT OF THE CROSS-VARIOGRAM CAN BE ESTIMATED WHEN $h = 0$

CO-KRIGING VARIANCE:

AFTER TAKING EXPECTATIONS WE FIND:

$$\text{MIN } C_{11}(0) - 2\sum w_{1i} C_{11}(s_0, s_i)$$

$$- 2\sum w_{2i} C_{12}(s_0, s_i)$$
$$+ \sum\sum w_{1i} w_{1k} C_{11}(s_i, s_k)$$
$$+ \sum\sum w_{2i} w_{2k} C_{22}(s_i, s_k)$$
$$+ 2\sum\sum w_{1i} w_{2j} C_{12}(s_i, s_j) \qquad (1)$$

$$- 2\lambda_1 \left[\sum w_{1i} - 1\right] - 2\lambda_2 \left[\sum w_{2i} - 0\right]$$

DERIVATIVE WRT w_{1i}:

$$-2 C_{11}(s_0, s_i) + 2\sum w_{ik} C_{11}(s_i, s_k) + 2\sum w_{2j} C_{12}(s_i, s_j) - 2\lambda_1 = 0 \qquad (2)$$

DERIVATIVE WRT w_{2i}:

$$2 C_{12}(s_0, s_i) + 2\sum w_{2k} C_{22}(s_i, s_k) + 2\sum w_{1i} C_{12}(s_i, s_j) - 2\lambda_2 = 0 \qquad (3)$$

MULTIPLY (2) BY w_{1i} AND SUM OVER $i = 1, \ldots, n$ TO GET:

$$-\sum w_{1i} C_{11}(s_0, s_i) + \sum\sum w_{1i} w_{1k} C_{11}(s_i, s_k) \qquad (4)$$
$$+ \sum\sum w_{1i} w_{2j} C_{12}(s_i, s_j) - \lambda_1 = 0$$

MULTIPLY (3) BY w_{2i} AND SUM OVER $i = 1, \ldots, n$ TO GET:

$$-\sum w_{2i} C_{12}(s_0, s_i) + \sum\sum w_{2i} w_{2k} C_{22}(s_i, s_k)$$
$$+ \sum\sum w_{1i} w_{2j} C_{12}(s_i, s_j) = 0 \qquad (5)$$

SUBSTITUTE (4) AND (5) INTO (1) TO GET:

$$\sigma^2(s_0) = C_{11}(0) - \sum w_{1i}\, C_{11}(s_0, s_i) - \sum w_{2i}\, C_{12}(s_0, s_i) + \lambda_1$$

OR

$$\sigma^2(s_0) = C_{11}(0) - \sum_{j=1}^{2} \sum_{i=1}^{n} w_{ji}\, C_{ji}(s_0, s_i) + \lambda_1$$