Problem 1
Consider three locations on the unit line, denoted \( s_1, s_2, \) and \( s_3 \), which are located at coordinates 1, 2, and 3 respectively. Variable \( Z_1 \) is observed at location \( s_1 \) and \( s_2 \) with values 21 and 23 respectively. The collocated variable \( Z_2 \) is observed at \( s_1, s_2, \) and \( s_3 \) with values 5, 6, and 6 respectively. The exponential covariance functions are known and are given as follows:

- Target variable covariance function:
  \[ c_1(h) = c_0 \delta(h) + c_1 \exp(-h/\alpha), \]
  \( c_0 = 1, c_1 = 3, \alpha = 0.5 \).

- Collocated variable covariance function:
  \[ c_2(h) = c_0 \delta(h) + c_1 \exp(-h/\alpha), \]
  \( c_0 = 0.3, c_1 = 1.7, \alpha = 0.5 \).

- Cross covariance function:
  \[ c_{12}(h) = c_0 \delta(h) + c_1 \exp(-h/\alpha), \]
  \( c_0 = 0.4, c_1 = 1.9, \alpha = 0.5 \).

Note: Use \( \delta(h) = 1 \) if \( h = 0 \) and \( \delta(h) = 0 \) if \( h \neq 0 \).

Use the information above to predict the target variable at coordinates 1.5, 2.5, and 3.5 and its variance using ordinary kriging.

Problem 2
Access the following data:

\[ a <- \text{read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/final_q2_w19.txt", header=TRUE)} \]

We will model \( Z \) as log-normal. That is, we first predict \( Y \) where \( Y(s) = \log(Z(s)) \) and use the results to predict \( Z(s) \). To model \( Y(s) \) use the exponential covariance function with \( \alpha = 0.5, c_1 = 1, \) and \( c_0 = 0 \). Use simple log-normal kriging with \( E(Y) = \mu = 5 \) to predict \( Z \) on the 10 \times 10 \) grid. Construct the raster map using the predicted values.

Problem 3
Use the spectral decomposition method to simulate data within the state of Colorado. Assume that the data is a realization of \( N(\mu, \Sigma) \).

1. For a covariance function use the Gaussian model with zero nugget and sill and range parameters of your choice.
2. Plot the covariance function.
3. Add the points on the map and create a circle plot of the data.
Problem 4
Consider the 4-point layout given on the graph below. The \( z(s_1), z(s_2), z(s_3), z(s_4) \) are the observed values of the \( Z \) process which is described by the exponential semivariogram \( \gamma(h) = c_0 + c_1(1 - e^{-\frac{h}{\alpha}}) \), with \( c_0 = 0, c_1 = 3.5, \alpha = 4.5 \). Our goal is to predict the value \( z(s_0) \) at location \( s_0 \). The coordinates of these 5 points are:

<table>
<thead>
<tr>
<th>( s_i )</th>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( z(s_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>1</td>
<td>2</td>
<td>??</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>1</td>
<td>1</td>
<td>513</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>1</td>
<td>3</td>
<td>531</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>2</td>
<td>1</td>
<td>516</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>2</td>
<td>3</td>
<td>537</td>
</tr>
</tbody>
</table>

Answer the following questions:

a. Compute the distance matrix of these 5 points.

b. Ordinary kriging: Using the variogram, compute the matrix \( \Gamma \) and the vector \( \gamma \) needed for the calculation of the ordinary kriging weights. Note: The matrix and vector here, are the ones that incorporate the unbiasedness constraint.

c. Simple kriging: Using the covariance, compute the matrix \( C \) and the vector \( c \) needed for the calculation of the simple kriging weights.

d. Universal kriging: Assume a linear trend as a function of the coordinates \( x \) an \( y \). Using the variogram, compute the matrix \( \Gamma_1 \) and the vector \( \gamma_1 \) needed for the calculation of the universal kriging weights. Note: The matrix and vector here, are the ones that incorporate the unbiasedness constraints.