Problem 1  (25 points)
Consider the coal-ash data:

```r
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/coal_ash.txt", header=TRUE)
```

There are 208 measurements on coal-ash. The first six rows of the data set and the spatial locations of the data points are given below:

```r
> head(a)
   x y coalash
 1 1 14 10.21
 2 1 15  9.92
 3 1 16 11.17
 4 2  8 10.01
 5 2 10 11.15
 6 2 11 11.31
```

Things to do using only `gstat`:

a. Compute the sample variogram.

b. Fit a model to the sample variogram.

c. Create a grid (use `by=0.1`).

d. Perform ordinary kriging predictions.

e. Construct the raster map using the predicted values from (d).
Problem 2 (25 points) - Due on Monday, 03 March

Consider the 4-point layout given on the graph below. The \( z(s_1), z(s_2), z(s_3), z(s_4) \) are the observed values of the \( Z \) process which is described by the exponential semivariogram \( \gamma(h) = c_0 + c_1(1 - e^{-\frac{h}{\alpha}}) \), with \( c_0 = 0, c_1 = 3.5, \alpha = 4.5 \). Our goal is to predict the value \( z(s_0) \) at location \( s_0 \). The coordinates of these 5 points are:

<table>
<thead>
<tr>
<th>( s_i )</th>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( z(s_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>1</td>
<td>2</td>
<td>???</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>1</td>
<td>1</td>
<td>513</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>1</td>
<td>3</td>
<td>531</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>2</td>
<td>1</td>
<td>516</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>2</td>
<td>3</td>
<td>537</td>
</tr>
</tbody>
</table>

Answer the following questions:

a. Compute the distance matrix of these 5 points.

b. Ordinary kriging: Using the variogram, compute the matrix \( \Gamma \) and the vector \( \gamma \) needed for the calculation of the ordinary kriging weights. You can also use the covariance function and solve it using matrix \( C \) and vector \( c \).

c. Simple kriging: Using the covariance, compute the matrix \( C \) and the vector \( c \) needed for the calculation of the simple kriging weights.

d. Universal kriging: Assume a linear trend as a function of the coordinates \( x \) an \( y \). Using the variogram, compute the matrix \( \Gamma_1 \) and the vector \( \gamma_1 \) needed for the calculation of the universal kriging weights.

e. Use \texttt{geoR} and the function \texttt{ks.line} to estimate the value of \( Z \) at \( s_0 \) and its variance with ordinary kriging.

f. Use \texttt{geoR} and the function \texttt{krige.conv} to estimate the value of \( Z \) at \( s_0 \) and its variance with universal kriging.
Problem 3  (25 points)
Consider the points in the figure below. Locations $s_1, s_2, s_3$ are the data points and location $s_0$ is the location of the point to be predicted. Assume the spherical variogram with $c_0 = 0, c_1 = 1, \alpha = 50$. If $d = 10$ set up the ordinary kriging system and compute:

a. The weights $w_1, w_2, w_3$.

b. The ordinary kriging variance.
**Problem 4 (25 points)**

In general, kriging assigns larger weights to the closer points than data farther away from the point to be predicted. Let’s examine here what happens if the point to be predicted and two data points are in line as shown in the figure below.

\[ s_2 \quad h_1 = 5 \quad s_1 \quad h_2 = 7 \quad s_0 \]

This is called the “screening” effect of kriging. It means that the weight of a data point which is screened by another data point is reduced dramatically when the two points are in line with the point to be predicted.

Use simple kriging and assume exponential covariance function with \( c_0 = 0, c_1 = 10, \) and \( \alpha = 40 \) to find the weights \( w_1 \) and \( w_2 \).

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**For home - due on Monday, 03 March**

Do this problem again as follows: Assume that the distance from \( s_1 \) to \( s_2 \) is \( h_1 \) and the the distance from \( s_0 \) to \( s_1 \) is \( h_2 \). So the distance from \( s_0 \) to \( s_2 \) is \( h_1 + h_2 \). Use the exponential covariance function with sill \( c_1 \) and range \( \alpha \). You will need to find the inverse of a \( 2 \times 2 \) matrix in order to compute the weights. What do you find?

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**For home - due on Monday, 03 March**

Consider ordinary lognormal kriging (we work with \( Y(s_0) = \log(Z(s_0)) \)). Find the variance of the prediction error in terms of the original scale of the data, i.e. find \( E(Z(s_0) - \hat{Z}(s_0))^2 \).