University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Homework 2

EXERCISE 1

Please refer to the prediction problem for the non-i.i.d. case we discussed in class on Friday, 01/10/20.

a. Show that $\hat{Y}_0 = \hat{\mu} + \mathbf{c}'\mathbf{\Sigma^{-1}}(\mathbf{Y} - \hat{\mu}\mathbf{1})$, where $\hat{\mu} = \frac{\mathbf{1'\Sigma^{-1}Y}}{\mathbf{1'\Sigma^{-1}1}}$.

b. Show that the variance of the error of prediction is:

$$\mathrm{var}(Y_0 - \hat{Y}_0) = \sigma^2 - \mathbf{c}'\mathbf{\Sigma^{-1}}\mathbf{c} + \frac{(1 - \mathbf{1}'\mathbf{\Sigma^{-1}}\mathbf{c})^2}{\mathbf{1}'\mathbf{\Sigma^{-1}}\mathbf{1}}.$$

EXERCISE 2

Suppose $Y_i = \mu + \epsilon_i$, with $E(\epsilon_i) = 0, \mathrm{var}((Y_i) = \sigma^2$, and $\mathrm{cov}(Y_i, Y_j) = \sigma^2\rho^{|i-j|}$. Assume that $\rho > 0$. Draw the power function in R for testing $H_0 : \mu = \mu_0$ against $H_a : \mu > \mu_o$ when we use the test statistic $Z = \frac{\bar{Y} - \mu_0}{\mathrm{sd}(\bar{Y})}$. On the same plot, draw also the power function when $\rho = 0$ when testing the same hypothesis.