EXERCISE 1
The robust estimator of the variogram (Cressie and Hawkins (1980)) given by
\[
2\hat{\gamma}(h) = \left\{ \frac{1}{N(h)} \sum_{N(h)} |Z(s_i) - Z(s_j)|^\frac{1}{4} \right\}^4 \text{ uses the transformation } \left[ \frac{(Z(s + h) - Z(s))}{\sqrt{2\gamma(h)}} \right]^2 \right]^{\frac{1}{2}}.
\]

Consider now the class of power transformations
\[
\left[ \frac{(Z(s + h) - Z(s))}{\sqrt{2\gamma(h)}} \right]^2 \right]^{\lambda}.
\]

a. Find the mean and variance of a $[\chi^2]^{\lambda}$, where $\lambda = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.
b. Let $X \sim \chi^2$. Find and plot the pdf of $Y = X^{\lambda}$, where $\lambda = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.
c. Find an estimator of the variogram when the transformations
\[
\left[ \frac{(Z(s + h) - Z(s))}{\sqrt{2\gamma(h)}} \right]^2 \right]^{\frac{1}{2}} \text{ and } \left[ \frac{(Z(s + h) - Z(s))}{\sqrt{2\gamma(h)}} \right]^2 \right]^{\frac{1}{2}} \text{ are considered.}
\]
(See class notes for details).

EXERCISE 2
Access the soil data:
\[
a <- \text{read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/soil.txt", header=TRUE)}
\]
Answer the following questions:

a. Convert a into a geodata object using geoR.
b. Compute and plot the empirical semivariogram up to a maximum distance of 1800.
c. Fit by eye a spherical theoretical semivariogram and add the plot on the empirical variogram.
d. Fit the spherical semivariogram using the default weights and add it on the plot of (c).
e. Fit the spherical semivariogram using Cressie’s weights and add it on the plot of (c).
f. Fit the spherical semivariogram using equal weights and add it on the plot of (c).
g. Fit the spherical semivariogram using MML and add it on the plot of (c).
EXERCISE 3
Access the jura data:

```r
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/jura.txt", header=TRUE)
```

Answer the following questions:

a. Create a data frame using the first two columns (coordinates) and variable Cd. Convert this data frame into a geodata object using geoR and compute the maximum distance in your data.

b. Compute and plot the empirical semivariogram up to a maximum distance of 3.

c. Fit by eye a spherical theoretical semivariogram and add the plot on the empirical semivariogram.

d. Indicator semivariograms: Transform your data into 0, 1 data based on some threshold. Form example

```r
I <- ifelse(a$Cd >= 1, 1, 0)
```

Your variable now is I. Construct a data frame using the first two columns (coordinates) and this indicator variable I. Compute and plot the empirical semivariogram up to a maximum distance of 5.5, fit by eye a gaussian theoretical semivariogram, and add the plot on the empirical variogram. Do the same using maximum distance of 2.

EXERCISE 4
Consider the spatial locations for the ozone monitoring stations in California. You can access them here:

```r
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/coords.txt", header=TRUE)
```

Answer the following questions:

a. Using the Cholesky decomposition method and assuming multivariate normal data with mean \( \mu = 0.10 \) and variance covariance matrix \( \Sigma \), simulate manually the following isotropic process: Exponential model with \( c_0 = 0, c_1 = 0.05, \alpha = 2.8 \).

b. Compute the sample semivariogram and fit the exponential model to it. On the same graph plot the theoretical model used in (a).