# University of California, Los Angeles
# Department of Statistics

**Statistics C173/C273** **Instructor: Nicolas Christou**

## Homework 4

**EXERCISE 1**
The robust estimator of of the variogram (Cressie and Hawkins (1980)) given by

$$2\bar{\gamma}(h) = \frac{\left\{\frac{1}{N(h)}\sum_{N(h)}|Z(s_i)-Z(s_j)|^{\frac{1}{2}}\right\}^4}{0.457+\frac{0.494}{N(h)}} \text{ uses the transformation } \left[\left(\frac{Z(s+h)-Z(s)}{\sqrt{2\gamma(h)}}\right)^2\right]^{\frac{1}{4}}.$$

Consider now the class of power transformations

$$\left[\left(\frac{Z(s+h)-Z(s)}{\sqrt{2\gamma(h)}}\right)^2\right]^{\lambda}.$$

a. Find the mean and variance of a $[\chi_1^2]^\lambda$, where $\lambda = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.

b. Let $X \sim \chi_1^2$. Find and plot the pdf of $Y = X^\lambda$, where $\lambda = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.

c. Find an estimator of the variogram when the transformations

$$\left[\left(\frac{Z(s+h)-Z(s)}{\sqrt{2\gamma(h)}}\right)^2\right]^{\frac{1}{2}} \text{ and } \left[\left(\frac{Z(s+h)-Z(s)}{\sqrt{2\gamma(h)}}\right)^2\right]^{\frac{1}{3}} \text{ are considered.}$$

(See class notes for details).

**EXERCISE 2**
Consider the Moran's test statistic $I$ for spatial autocorrelation. If we use the model $\mathbf{Z} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ then the test statistic is given by $I = \frac{n}{S_0}\frac{\mathbf{e}'\mathbf{W}\mathbf{e}}{\mathbf{e}'\mathbf{e}}$. Using this formula, show that when the model is $\mathbf{Z} = \mu\mathbf{1} + \boldsymbol{\epsilon}$ we get $E(I) = -\frac{1}{n-1}$. Find and expression for $E(I)$ when that when $k = 1$ predictor (simple regression, $y = \beta_0 + \beta_1 x_i + \epsilon_i$).