Exercise 1
Use the variable zinc of the soil data to perform cross validation with gstat and geoR.

a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/soil.txt", 
    header=TRUE)

a. Using gstat:
   1. Split the data into two parts (one for modeling and one for cross validation). Create a gstat object and use it to compute the sample variogram and fit the spherical and exponential variograms to it. Predict the points of the cross validation part of the data set and compare the prediction sum of squares (PRESS) for each variogram.
   2. Delete one point at a time and use the remaining n − 1 points to predict it. Compare the PRESS for the two variograms.

b. Using geoR: Create a data frame with x, y, zinc. Convert the data frame into a geodata object and use it to compute the sample variogram. Fit the spherical and exponential variograms to it and finally use the xvalid function with reest=TRUE to compare the two PRESS.

c. Based on your analysis above, which model variogram will choose?

d. Please submit all the R commands and results from questions (a) and (b).

Exercise 2
Access the elevation data (see also homework 3, exercise 3):
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/elevation_data.txt", header=TRUE)


a. Use geoR or gstat to perform ordinary kriging on a dense grid of your choice.

b. Use geoR or gstat to perform universal kriging on the same grid of part (a). Would you choose ordinary or universal kriging?

c. Plot the raster maps of the predicted values and their kriging variances.

Exercise 3
The following data give the location (x, y coordinates) and the calcium content at depth 0-20 cm (ca20), for each data point. There are 178 data points. Please access the data at:
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/soil_ca_data.txt", header=TRUE)

a. Create a grid for spatial predictions (by=10).

b. Create a gstat object assuming that there is a linear trend in the data (on the coordinates x, y).

c. Plot the semivariogram up to a maximum distance of 510 m.

d. Fit the spherical semivariogram to the sample semivariogram above using Cressie’s weights.

e. Perform universal kriging (linear trend on the coordinates).

f. Collapse the vector of the predicted values into a matrix and use the image function to create a raster map. Add contours to the raster map.

g. Collapse the vector of the variances of the predicted values into a matrix and use the image function to create a raster map. Add contours to the raster map.

Exercise 4
Consider universal kriging. In matrix/vector form universal kriging minimizes

\[ C(0) - 2\epsilon'w + w'\Sigma w, \]

subject to the set of constraints \( X'w = x \). Find explicit solutions for \( w \) and \( \lambda \), where \( w = (w_1, w_2, \ldots, w_n)' \) and \( \lambda = (\lambda_0, \lambda_1, \ldots, \lambda_k)' \) is the vector of the Lagrange multipliers. Show that \( \hat{Z}(s_0) = x_0'\hat{\beta}_{gls} + c'\Sigma^{-1}(Z - X\hat{\beta}_{gls}). \)

Exercise 5
Show that using the simple kriging weights and the generalized least squares estimate of \( \beta \) we obtain the universal kriging weights. The generalized least squares estimate of \( \beta \) is given by \( \hat{\beta}_{gls} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Z. \)
Exercise 6
Consider the one-dimensional kriging problem as shown on the next figure:

Suppose the covariance function of range \( \alpha \) is given by

\[
C(h) = \begin{cases} 
  c_1(1 - \frac{h}{\alpha}), & h \leq \alpha \\
  0, & h > \alpha 
\end{cases}
\]

We want to predict point \( s_0 \) at \( s_0 = 0 \) using points \( s_1 = -\frac{\alpha}{2}, \ s_2 = \frac{\alpha}{2} \) and \( s_3 = \frac{\alpha}{2} + \alpha f \), with \( 0 < f \leq 1 \). Show that when \( 0 < f \leq \frac{1}{2} \) the ordinary kriging weights are \( w_1 = \frac{2}{\pi f} \), \( w_2 = \frac{3 - f}{\pi f} \), and \( w_3 = -\frac{2}{\pi f} \).