University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Homework 7

Exercise 1
Answer the following questions:

a. Consider simple lognormal kriging. We discussed in class that the unbiased predictor of $Z(s_0)$ is given by $\check{Z}(s_0) = e^{\hat{Y}(s_0)+\frac{1}{2}\sigma^2_{SK}}$. Find $E[Z(s_0) - \check{Z}(s_0)]^2$.

b. Consider ordinary lognormal kriging. Note: We predict $Y(s_0)$, where $Y(s_0) = ln[Z(s_0)]$. Show that the unbiased predictor of $Z(s_0)$ is given by $\check{Z}(s_0) = e^{\frac{1}{2}\sigma^2_{OK}-\lambda+\hat{Y}(s_0)}$.

Exercise 2
Show that kriging is an exact interpolator. It means that if the location of the point to be predicted is the same with the location of one of the observed values the predicted value will be equal to the observed value.

Problem 3
Consider the 4-point layout given on the graph below. The $z(s_1), z(s_2), z(s_3), z(s_4)$ are the observed values of the Z process which is described by the exponential semivariogram $\gamma(h) = c_0 + c_1(1 - e^{-\frac{h}{\alpha}})$, with $c_0 = 0, c_1 = 3.5, \alpha = 4.5$. Our goal is to predict the value $Z(s_0)$ at location s_0. The coordinates of these 5 points are:

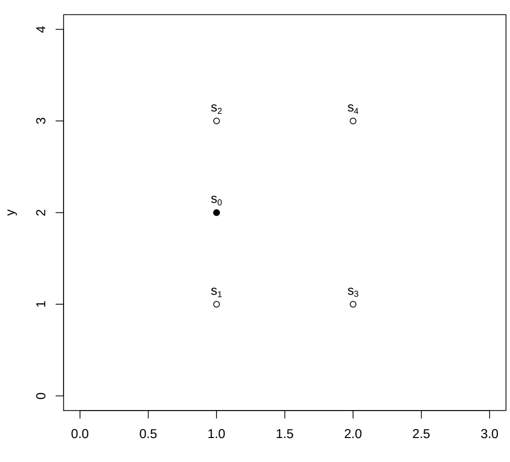
s_i	x_i	y_i	$z(s_i)$
s_0	1	2	???
s_1	1	1	513
s_2	1	3	531
s_3	2	1	516
s_4	2	3	537

a. Compute the distance matrix of these 5 points.

b. Ordinary kriging: Using the variogram, compute the matrix $\boldsymbol{\Gamma}$ and the vector $\boldsymbol{\gamma}$ needed for the calculation of the ordinary kriging weights.

c. Simple kriging: Using the covariance, compute the matrix $\boldsymbol{C}$ and the vector $\boldsymbol{c}$ needed for the calculation of the simple kriging weights.

d. Use the function `ksline` of `geoR` to predict the value of $Z(s_0)$ at s_0 and its variance with ordinary kriging.



Exercise 4
Consider the coal-ash data:

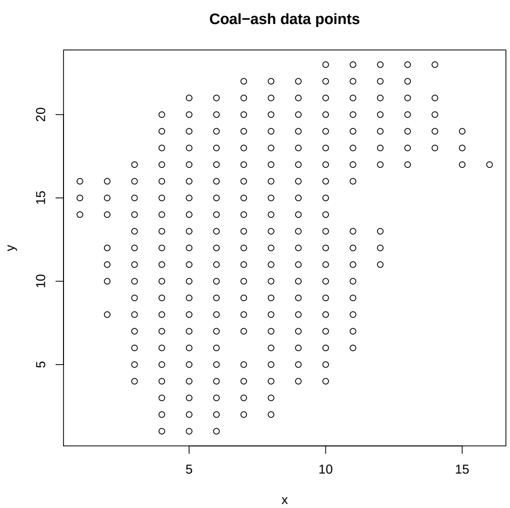
```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/
coal_ash.txt", header=TRUE)
```

There are 208 measurements on coal-ash. The first six rows of the data set and the spatial locations of the data points are given below:

```
> head(a)
  x  y coalash
1 1 14   10.21
2 1 15    9.92
3 1 16   11.17
4 2  8   10.01
5 2 10   11.15
6 2 11   11.31
```

For this problem use `gstat` to answer the following questions:

a. Compute the sample variogram.
b. Fit a model to the sample variogram.
c. Create a grid (use `by=0.1`).
d. Perform ordinary kriging predictions.
e. Construct the raster map using the predicted values from (d).



Exercise 5
Consider the points in the figure below. Locations s_1, s_2, s_3 are the data points and location s_0 is the location of the point to be predicted. Assume the spherical variogram with $c_0 = 0, c_1 = 1, \alpha = 50$. If $d = 10$ set up the ordinary kriging system and compute:

a. The weights w_1, w_2, w_3.
b. The ordinary kriging variance.