Kriging revisited

We observe \( Z = (Z(s_1), Z(s_2), \ldots, Z(s_n))^t \) and we want to predict \( Z(s_0) \).

**Theorem 1**

As discussed, kriging minimizes the mean square prediction error, \( MSE(\hat{Z}(s_0)) = E(Z(s_0) - \hat{Z}(s_0))^2 \). An important result is the following: \( MSE(\hat{Z}(s_0)) \) takes its minimum value when \( \hat{Z}(s_0) = E(Z(s_0)|Z) \).

**Theorem 2**

Suppose that \( Y, \mu, \) and \( \Sigma \) are partitioned as follows \( Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}, \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \), and \( Y \sim MVN(\mu, \Sigma) \). It can be shown that the conditional distribution of \( Y_1 \) given \( Y_2 \) is also multivariate normal, \( Y_1|Y_2 \sim MVN(\mu_{1|2}, \Sigma_{1|2}) \), where \( \mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(Y_2 - \mu_2) \), and \( \Sigma_{1|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \).

To apply this theorem in the spatial prediction problem assume the distribution of \( (Z(s_0), Z) \) is multivariate normal with mean vector \( \mu_1 \) and variance covariance matrix \( \begin{pmatrix} \sigma^2 & c' \\ c & C \end{pmatrix} \).

**Result**

Using the previous theorems, the predictor that minimizes the mean square prediction error (see Theorem 1) will be (see Theorem 2) \( \hat{Z}(s_0) = \mu + c'\Sigma^{-1}(Z - \mu_1) \), which is the simple kriging predictor. The prediction variance (also see Theorem 2) will be \( \sigma^2 - c'\Sigma^{-1}c = C(0) - c'\Sigma^{-1}c \), which is the simple kriging variance.