Exercise 1
These Jura data were collected by the Swiss Federal Institute of Technology at Lausanne. See Goovaerts, P. 1997, “Geostatistics for Natural Resources Evaluation”, Oxford University Press, New-York, 483 p. for more details. Data were recorded at 359 locations scattered in space (see figure below).

Concentrations of seven heavy metals (cadmium, cobalt, chromium, copper, nickel, lead, and zinc) in the topsoil were measured at each location. The type of land use and rock type was also recorded for each location. The data can be accessed here:

a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/jura.txt", header=TRUE)

> names(a)
[1] "x"  "y"  "Landuse" "Rock"  "Cd"
[6] "Co"  "Cr"  "Cu"  "Ni"  "Pb"
[11] "Zn"

The variables x, y are the coordinates. Landuse and Rock represent type of land use (forest, pasture, meadow, tillage) and rock type (Argovian, Kimmeridgian, Sequanina, Portlandian, and Quaternary). The other variables are concentrations in ppm of the following chemical elements:
Cd: Cadmium  
Co: Cobalt  
Cr: Chromium  
Cu: Copper  
Ni: Nickel  
Pb: Lead  
Zn: Zinc

Answer the following questions:

a. Construct histograms, boxplots, and empirical cumulative distribution functions for Cd, Co, Cr, Cu, Ni, Pb, Zn.
b. Compute the distance matrix (359 x 359).
c. Construct h-scatterplots using Cu.
Exercise 2

Let \( Z(s) \) be a stationary random function, and let \( \hat{Z}(s_0) = 0.5Z(s_1) + 0.2Z(s_2) + 0.2Z(s_3) + 0.1Z(s_4) \) be a weighted average of the four values \( Z(s_1), Z(s_2), Z(s_3), Z(s_4) \) as shown on the \( 2m \times 2m \) square below.

[Diagram of a 2x2 square with points labeled s1, s2, s3, s4, and s0]

Answer the following questions:

a. Use \( R \) to compute the distance matrix.

b. Assume that the spatial covariance is given by \( C(h) = 2.5e^{-\frac{h}{2}} \). Use \( R \) to compute the \( 4 \times 4 \) variance covariance matrix of the vector \( Z = \begin{pmatrix} Z(s_1) \\ Z(s_2) \\ Z(s_3) \\ Z(s_4) \end{pmatrix} \).

c. Use matrix and vector notation to compute \( \text{var}(Z(s_0)) \). Note: Write \( Z(s_0) = a'Z \), where \( a = \begin{pmatrix} 0.5 \\ 0.2 \\ 0.2 \\ 0.1 \end{pmatrix} \). Then use \( \text{var}(Z(s_0)) = a'\Sigma a \).