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Statistics C173/C273 **Instructor: Nicolas Christou**

Test statistics for lattices

A. Binary attribute

Use the BB and BW join counts.

$$BB = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} Z(s_i) Z(s_j)$$

$$BW = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} [Z(s_i) - Z(s_j)]^2$$

Assume sampling with replacement (binomial) with $P(Z(s_i) = 1) = p$ then under the hypothesis of no autocorrelation the mean and variance of BB and BW are as follows:

$$E[BB] = \frac{1}{2}p^2 \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}$$

$$\text{var}(BB) = \frac{1}{4}p^2(1-p)\left[S_1(1-p) + S_2 p\right]$$

$$E[BW] = p(1-p)\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}$$

$$\text{var}(BW) = S_1 p(1-p) + \frac{1}{4}S_2 p(1-p)[1 - 4p(1-p))$$

where,

$$S_1 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(w_{ij} + w_{ji})^2$$

$$S_2 = \sum_{i=1}^{n}\left[\sum_{j=1}^{n} w_{ij} + \sum_{j=1}^{n} w_{ji}\right]^2$$

B. Continuous attribute

Moran's and Geary's statistics

The distribution of Moran's and Geary's test statistics under normality: Assume $Z(s_1), \ldots, Z(s_n)$ follow $N(\mu, \sigma)$.

a. Moran's I statistic:

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij}(Z(s_i) - \bar{Z})(Z(s_j) - \bar{Z})}{\sum_{i=1}^n (Z(s_i) - \bar{Z})^2}.$$

with

$E(I) = -\frac{1}{n-1}$.

$E(I^2) = \frac{1}{(n-1)(n+1)S_0^2}(n^2 S_1 - nS_2 + 3S_0^2)$.

where,

$S_0 = \sum_{i=1}^n \sum_{j=1}^n w_{ij}$.

$S_1 = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (w_{ij} + w_{ji})^2$.

$S_2 = \sum_{i=1}^n \left(\sum_{j=1}^n w_{ij} + \sum_{j=1}^n w_{ji} \right)^2$.

b. Geary's c statistic:

$$c = \frac{n-1}{2S_0} \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij}(Z(s_i) - Z(s_j))^2}{\sum_{i=1}^n (Z(s_i) - \bar{Z})^2}.$$