University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Moran's and Geary's statistics

The distribution of Moran's and Geary's test statistics under normality: Assume $Z(s_1), \ldots, Z(s_n)$ follow $N(\mu, \sigma)$.

a. Moran's I statistic:

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij}(Z(s_i) - \bar{Z})(Z(s_j) - \bar{Z})}{\sum_{i=1}^n (Z(s_i) - \bar{Z})^2}.$$

with
$E(I) = -\frac{1}{n-1}$.
$E(I^2) = \frac{1}{(n-1)(n+1)S_0^2}(n^2 S_1 - nS_2 + 3S_0^2)$.
where,
$S_0 = \sum_{i=1}^n \sum_{j=1}^n w_{ij}$.
$S_1 = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (w_{ij} + w_{ji})^2$.
$S_2 = \sum_{i=1}^n \left(\sum_{j=1}^n w_{ij} + \sum_{j=1}^n w_{ji} \right)^2$.

b. Geary's c statistic:

$$c = \frac{n-1}{2S_0} \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij}(Z(s_i) - Z(s_j))^2}{\sum_{i=1}^n (Z(s_i) - \bar{Z})^2}.$$