

University of California, Los Angeles  
Department of Statistics

Statistics C173/C273

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Inverse of a partitioned matrix

If all inverses exist,

$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A}_{11}^{-1} + \mathbf{B}_{12}\mathbf{B}_{22}^{-1}\mathbf{B}_{21} & -\mathbf{B}_{12}\mathbf{B}_{22}^{-1} \\ -\mathbf{B}_{22}^{-1}\mathbf{B}_{21} & \mathbf{B}_{22}^{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_{11}^{-1} & -\mathbf{C}_{11}^{-1}\mathbf{C}_{12} \\ -\mathbf{C}_{21}\mathbf{C}_{11}^{-1} & \mathbf{A}_{22}^{-1} + \mathbf{C}_{21}\mathbf{C}_{11}^{-1}\mathbf{C}_{12} \end{pmatrix}$$

where

$$\mathbf{B}_{22} = \mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}$$

$$\mathbf{B}_{12} = \mathbf{A}_{11}^{-1}\mathbf{A}_{12}$$

$$\mathbf{B}_{21} = \mathbf{A}_{21}\mathbf{A}_{11}^{-1}$$

and

$$\mathbf{C}_{11} = \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}$$

$$\mathbf{C}_{12} = \mathbf{A}_{12}\mathbf{A}_{22}^{-1}$$

$$\mathbf{C}_{21} = \mathbf{A}_{22}^{-1}\mathbf{A}_{21}$$

Use these results to show that kriging is an exact interpolator.