University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 9 - Solutions

Exercise 1:
$\mu = 0.20$ $\sigma = 0.25$ $s = \$50$ $\Delta t = \frac{1}{52}$

a.

$$\begin{aligned}
\frac{\Delta s}{s} &= \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \\
\frac{\Delta s}{s} &= 0.20 \frac{1}{52} + 0.25 \epsilon \sqrt{\frac{1}{52}} \\
\frac{\Delta s}{s} &\sim N\left(\mu \Delta t, \sigma \sqrt{\Delta t}\right) \\
\frac{\Delta s}{s} &\sim N(0.00384615, 0.0346688)
\end{aligned}$$

b. See attached plot (next page) for the simulated path of the stock.

Exercise 2:
$\mu = 0.16$ $\sigma = 0.30$ $s = \$50$ $\Delta t = \frac{1}{365}$

$$\begin{aligned}
\frac{\Delta s}{s} &= \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \\
\Delta s &= \mu s \Delta t + \sigma s \epsilon \sqrt{\Delta t} \\
\Delta s &\sim N\left(\mu s \Delta t, \sigma s \sqrt{\Delta t}\right) \\
s + \Delta s &\sim N\left(s + \mu s \Delta t, \sigma s \sqrt{\Delta t}\right)
\end{aligned}$$

a. $E(s + \Delta s) = s + \mu s \Delta t = 50 + (0.16)50\frac{1}{365} = 50.0219178$

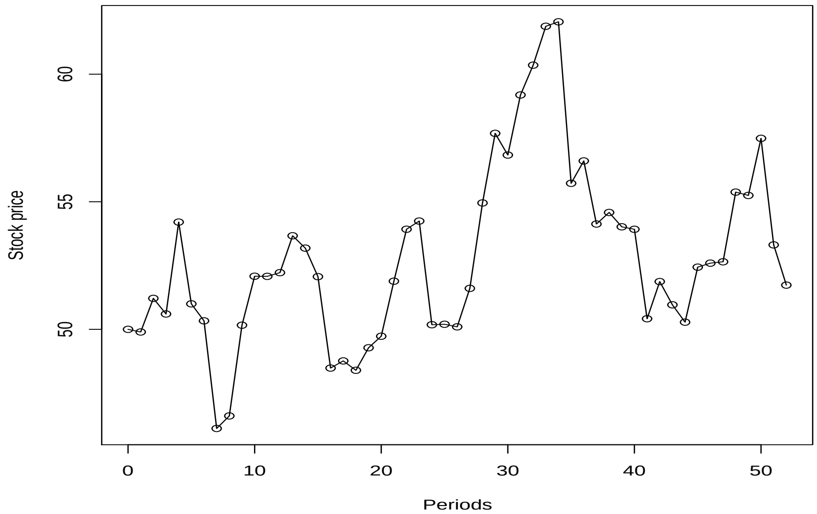
b. $SD(s + \Delta s) = \sigma s \sqrt{\Delta t} = (0.30)50\sqrt{\frac{1}{365}} = 0.785136$

Exercise 3:
Stock price follows Ito's process. $\mu = 0.16$ $\sigma = 0.35$ $S = \$38$

a. A European call option with an exercise price of \$40 will be exercised when the stock price exceeds \$40. Find $P(S > 40) = P(lnS > ln40) = P(lnS > 3.688879454)$.

$$\begin{aligned}
lnS_T &\sim N\left(lnS + (\mu - \frac{\sigma^2}{2})(T-t), \sigma\sqrt{T-t}\right) \\
lnS_{0.5} &\sim N\left(ln38 + \left(0.16 - \frac{0.35^2}{2}\right)(0.5), 0.35\sqrt{0.5}\right) \\
lnS_{0.5} &\sim N(3.68696116, 0.247487) \\
P\left(Z > \frac{ln40 - \mu}{\sigma}\right) &= P\left(Z > \frac{3.688879454 - 3.68696116}{0.247487}\right) \\
&= P(Z > 0.00775109) = 0.4969078
\end{aligned}$$

b. A European put option with an exercise price of \$40 will be exercise when the stock price does not exceed \$40. Find $P(S < 40) = 1 - P(S > 40) = 1 - 0.4969708 = 0.503092$.



Exercise 4:

$lnS_T \sim N\left(lnS + (\mu - \frac{\sigma^2}{2})(T-t), \sigma\sqrt{T-t}\right)$

$\mu^* = lnS + (\mu - \frac{\sigma^2}{2})(T-t) \qquad \sigma^* = \sigma\sqrt{T-t}$

$P(\mu^* - 1.96\sigma^* < lnS_T < \mu^* + 1.96\sigma^*) = 0.95$

$P(lnS + (\mu - \frac{\sigma^2}{2})(T-t) - 1.96\sigma\sqrt{T-t} < lnS_T < lnS + (\mu - \frac{\sigma^2}{2})(T-t) + 1.96\sigma\sqrt{T-t}) = 0.95$

$P(Se^{(\mu - \frac{\sigma^2}{2})(T-t) - 1.96\sigma\sqrt{T-t}} < S_T < Se^{(\mu - \frac{\sigma^2}{2})(T-t) + 1.96\sigma\sqrt{T-t}}) = 0.95$

$\mu = 0.10 \qquad \sigma = 0.15 \qquad S = \$40$

a. 95% interval for the price of the stock in 2 months:
$$\left(40e^{\left(0.10 - \frac{0.15^2}{2}\right)\left(\frac{1}{6}\right) - 1.96\times 0.15\times\sqrt{\frac{1}{6}}}, 40e^{\left(0.10 - \frac{0.15^2}{2}\right)\left(\frac{1}{6}\right) + 1.96\times 0.15\times\sqrt{\frac{1}{6}}}\right) = (36.0046, 45.7731)$$

b. $E(S_T) = Se^{\mu(T-t)} = 40e^{0.10\left(\frac{1}{6}\right)} = 40.6723$

c. $Var(S_T) = S^2e^{2\mu(T-t)}[e^{\sigma^2(T-t)} - 1] = 40^2e^{2\times 0.10\left(\frac{1}{6}\right)}[e^{0.15^2\left(\frac{1}{6}\right)} - 1] = 6.21502$
$SD(S_T) = \sqrt{Var(S_T)} = \sqrt{6.21502} = 2.49299$

Exercise 5:

Black-Scholes model: $S_0 = \$95 \qquad E = \$105 \qquad \sigma = 0.60 \qquad t = \frac{2}{3}$

$$\begin{aligned}
C &= S_0\Phi(d_1) - \left(\frac{E}{e^{rt}}\right)\Phi(d_2) \\
d_1 &= \frac{ln\left(\frac{S_0}{E}\right) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln\left(\frac{95}{105}\right) + (0.08 + \frac{1}{2}0.60^2)\frac{2}{3}}{0.60\sqrt{\frac{2}{3}}} = 0.149521 \\
d_2 &= \frac{ln\left(\frac{S_0}{E}\right) + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t} = 0.149521 - 0.489898 = -0.340377 \\
\Phi(d_1) &= \Phi(0.149521) = 0.5594287 \\
\Phi(d_2) &= \Phi(-0.340377) = 0.3667863 \\
C &= 95 \times 0.5594287 - \left(\frac{105}{e^{0.08\times\frac{2}{3}}}\right)0.3667863 = 16.6334
\end{aligned}$$