EXAM 2 SOLUTIONS

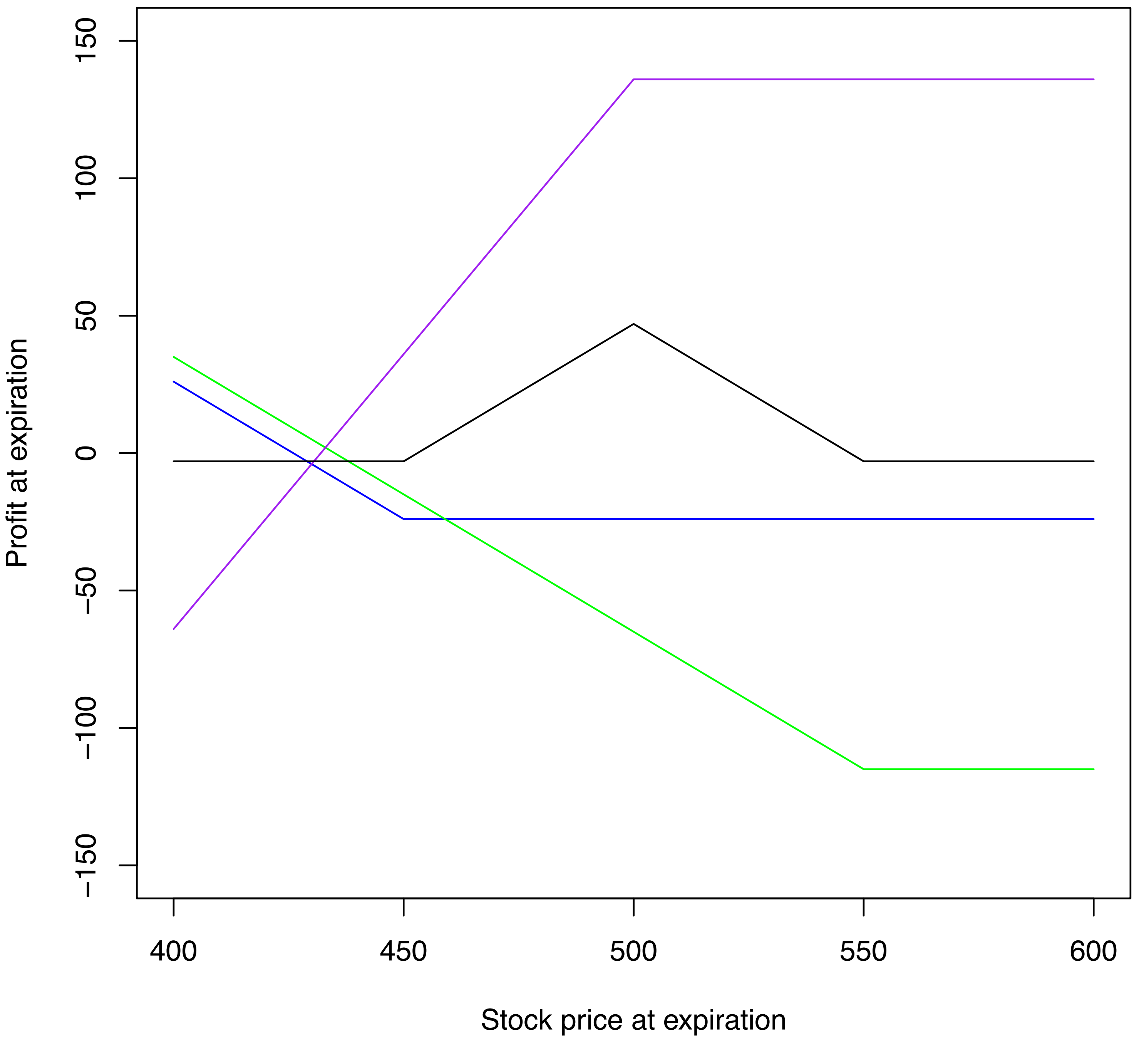
1. $E_1 = 450$, $E_2 = 500$, $E_3 = 550$
   $P_1 = 24$, $P_2 = 68$, $P_3 = 115$

BUY 1 — SELL 2 — BUY 1

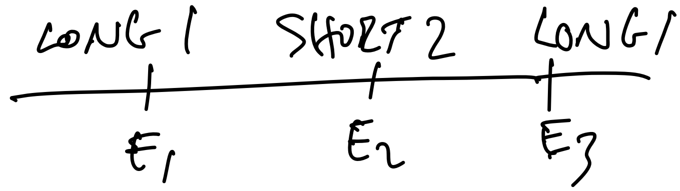
E_1 — E_2 — E_3

	LONG PUT E_1	LONG PUT E_3	SHORT PUT E_2	TOTAL	PROFIT
$S_T < E_1$	$450 - S_T$	$550 - S_T$	$-2(500 - S_T)$	0	-3
$E_1 < S_T < E_2$	0	$550 - S_T$	$-2[500 - S_T]$	$S_T - 450$	$S_T - 453$
$E_2 < S_T < E_3$	0	$550 - S_T$	0	$550 - S_T$	$447 - S_T$
$S_T > E_3$	0	0	0	0	-3

2.


150
100
50
0
−50
−100
−150
Profit at expiration
400
450
500
550
600
Stock price at expiration

3.



	LONG E_1	LONG E_3	SHORT E_2	TOTAL
$S_T < E_1$	0	0	0	0
$E_1 < S_T < E_2$	$S_T - E_1$	0	0	$S_T - E_1$
$E_2 < S_T < E_3$	$S_T - E_1$	0	$-2(S_T - E_2)$	$E_2 - E_1 - (S_T - E_2) \geq 0$
$S_T > E_3$	$S_T - E_1$	$S_T - E_3$	$-2(S_T - E_2)$	0

THE VALUE AT EXPIRATION IS ALWAYS ≥ 0

AT TIME 0 IT MUST BE $C_1 + C_3 - 2C_2 \geq 0$

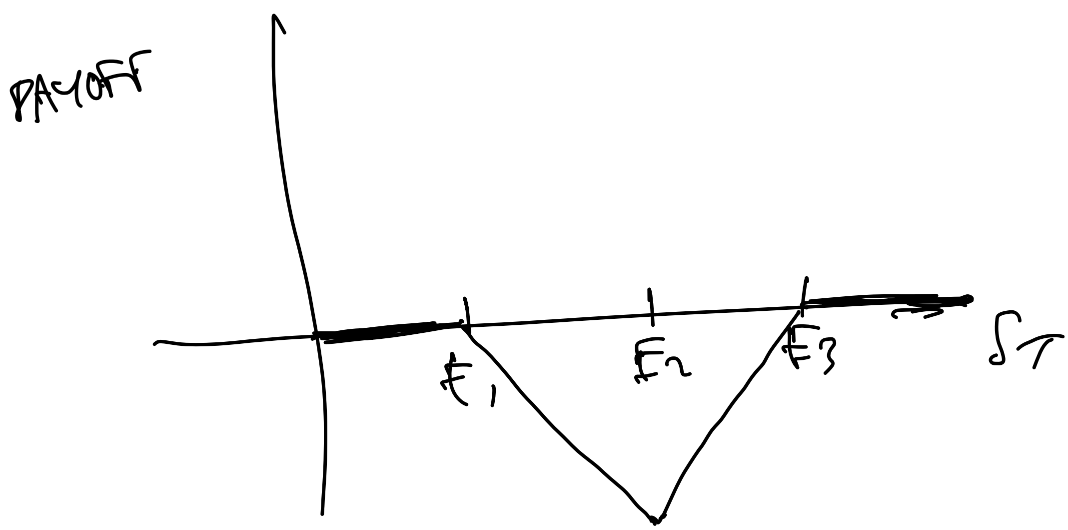
OR $C_2 \leq \frac{C_1 + C_3}{2}$

4.

E_1 E_2 E_3

SELL 1 BUY (2) SELL 1

	SHORT E_1	SHORT E_3	LONG E_2	TOTAL
$S_T < E_1$	0	0	0	0
$E_1 < S_T < E_2$	$E_1 - S_T$	0	0	$E_1 - S_T$
$E_2 < S_T < E_3$	$E_1 - S_T$	0	$2(S_T - E_2)$	$S_T - E_3$
$S_T > E_3$	$E_1 - S_T$	$E_3 - S_T$	$2(S_T - E_2)$	0



5. $P = 2.50 \quad S_0 = 47$

$t = 1$ MONTH , $E = 50$, $r = 6\%$

LOWER BOUND: $P \geq E e^{-rt} - S_0$

$$P \geq 50 e^{-0.06 \frac{1}{12}} - 47 = 2.75$$

THEREFORE $P = 2.50$ IS UNDERPRICED

$\left.\begin{array}{l}\text{BUY PUT}\\ \text{BUY STOCK}\end{array}\right\}$ BORROW $47 + 2.50 = 49.50$

IN 1 MONTH WE MUST RETURN

$$49.50 \, e^{0.06 \frac{1}{12}} = 49.75$$

- IF $S_T < 50 \rightarrow$ SELL AT $50 \rightarrow +0.25$
  (EXERCISE PUT OPTION

- IF $S_T > 50$, SAY 52
  THEN SELL AT $52 \rightarrow 2.25$

6. POSITION 2,2 :

$$1 + \sum_{i \in B} \frac{b_i^2}{\sigma_{\epsilon i}^2}\left[\sigma_{c2}^2 + b^2 \sigma_m^2\right]$$

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \bullet & \cdot & \cdot & \cdot \\ \bullet & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

POSITION 3,1

$$b_3 b_1 \sigma_m^2 \sum_{i \in C} \frac{b_i^2}{\sigma_{\epsilon i}^2}$$

7. 1. $\Phi_1 = \sum_{i=1}^{6} z_i \sigma_i$ (GROUP 1)

2. $C_1^* = \sum_{g=1}^{5} \rho_{1g} \Phi_g$

3. $z_1 = \dfrac{1}{\sigma_1(1-\rho_{11})}\left(\dfrac{\bar{R}_1 - R_F}{\sigma_1} - C_1^*\right)$

4. $\dfrac{N_5 \rho_{54}}{1-\rho_{55}}$

5. $\displaystyle\sum_{i=1}^{6} \frac{\bar{R}_i - R_F}{\sigma_i (1-\rho_{11})}$

8.

$\bar{R}_i$	$\bar{R}_i - R_F$	σ_i	$\frac{\bar{R}_i - R_F}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum \frac{\bar{R}_i - R_F}{\sigma_i}$	C_i
0.03284	0.02284	0.0762	0.2997	0.503	0.2997	0.151
0.01753	0.00753	0.0436	0.1643	0.335	0.464	0.155

$$C^* = 0.155$$

$$Z_1 = \frac{1}{\sigma_1(1-\rho)}\left(\frac{\bar{R}_1 - R_F}{\sigma_1} - C^*\right)$$

$$= \frac{1}{0.0762(1-0.503)}\left(0.2997 - 0.155\right) = 3.82$$

$$Z_2 = \frac{1}{\sigma_2(1-\rho)}\left(\frac{\bar{R}_2 - R_F}{\sigma_2} - C^*\right)$$

$$= \frac{1}{0.0426(1-0.503)}\left(0.1643 - 0.155\right) = 0.44$$

$$\therefore X_1 = \frac{Z_1}{Z_1+Z_2} = 0.897, \quad X_2 = 0.103$$

$$\left.\begin{array}{l} 9.\quad \hat{\alpha}_0 = \bar{R}_i \to \text{RESIDUALS } R_{it} - \bar{R}_i \\ \quad\;\; \hat{\gamma}_0 = \bar{R}_m \to \text{RESIDUALS } R_{mt} - \bar{R}_m \end{array}\right\}$$

REGRESS $R_{it} - \bar{R}_i$ ON $R_{mt} - \bar{R}_m$

$$R_{it} - \bar{R}_i = \delta_0 + \delta_1 \left(R_{mt} - \bar{R}_m\right) + \varepsilon_{it}$$

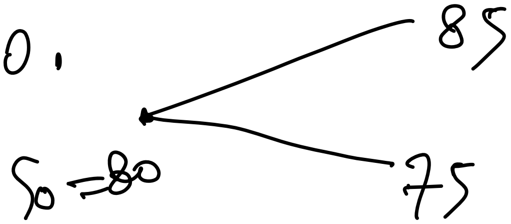
$$\hat{\delta}_1 = \frac{\sum \left(R_{mt} - \bar{R}_m\right)\left(R_{it} - \bar{R}_i\right)}{\sum \left(R_{mt} - \bar{R}_m\right)^2}$$

SAME AS BETA

NOTE: $\sum \left(R_{mt} - \bar{R}_m\right) = 0$

$\sum \left(R_{it} - \bar{R}_i\right) = 0$

10.



$r = 5\%$

$E = 80$

$t = 4$ MONTHS

	$t = 0$	$t=1$: $S = 75$	$S = 85$
BUY PUT	$-P$	$+5$	0
BUY α SHARES	-80α	75α	85α

$$75\alpha + 5 = 85\alpha$$

$$\alpha = \tfrac{1}{2}$$

PAYOFF IS 42.5

WITH NO ARBITRAGE OPPORTUNITIES
SINCE 42.5 IS RISKLES:

$$p + 80\tfrac{1}{2} = 42.5\, e^{-0.05 \frac{4}{12}}$$

$$p = 42.5\, e^{-0.05 \frac{4}{12}} - 40 = 1.798$$

11. RISK NEUTRAL VALUATION

$$80e^{0.05\frac{4}{12}} = p85 + (1-p)75$$

$$80e^{0.05\frac{4}{12}} - 75 = 10p$$

$$p = 0.6345 \quad \text{AND} \quad 1-p = 0.3655$$

$$5(0.3655)e^{-0.05\frac{4}{12}} = 1.798$$

$P =$

12.

	$t=0$	$t=1$: $S=75$	$S=85$
SELL CALL	$+C$	0	-5
BUY α SHARES	-80α	75α	85α
		75α	

$$75\alpha = 85\alpha - 5$$

$$\alpha = \frac{1}{2}$$

$$\text{PAYOFF} = 37.5$$

$$-C + 80\tfrac{1}{2} = 37.5e^{-0.05\frac{4}{12}} \quad \therefore C = 3.120$$

RISK NEUTRAL VALUATION

$$80e^{0.05\frac{4}{12}} = p85 + (1-p)75$$

$$p = 0.6345$$

$$C = 5(0.6345)e^{-0.05\frac{4}{12}} = 3.120$$

PUT CALL PARITY

$$P + S_0 = C + Ee^{-rt}$$

$$P + S_0 = 1.798 + 80 = 81.798$$

$$C + Ee^{-rt} = 3.120 + 80e^{-0.05\frac{4}{12}} = 81.798$$

13. (1). WHAT IS THE PAYOFF FOR THE BUYER OF A CALL OPTION?

(2) WHAT IS THE PAYOFF FOR THE SELLER OF A PUT OPTION?

(3) WHAT IS THE PAYOFF FOR THE SELLER OF A CALL OPTION?

(4) WHAT IS THE PAYOFF FOR THE BUYER OF PUT OPTION?

(5). WHAT IS PUT CALL PARITY?

14. $P_1 + S_0 = C_1 + E_1 e^{-rt}$

$P_2 + S_0 = C_2 + E_2 e^{-rt}$

$P_3 + S_0 = C_3 + E_3 e^{-rt}$

$$C_1 + C_3 - 2C_2 = \left(P_1 + S_0 - E_1 e^{-rt}\right) + \left(P_3 + S_0 - E_3 e^{-rt}\right) - 2\left(P_2 + S_0 - E_2 e^{-rt}\right)$$

$$= P_1 + P_3 - 2P_2 + 2S_0 - 2S_0 - E_1 e^{-rt} - E_3 e^{-rt} + 2E_2 e^{-rt}$$

$2E_2 = E_1 + E_3$

$$= P_1 + P_3 - 2P_2$$

15. $C^* = 0.016074557$

$COL1 = 5.6601546$

$COL4 = 48.06038$